

Parametric analysis of dynamic wave-seabed interaction

Análisis paramétrico de la interacción dinámica oleaje-suelo marino

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Abstract

In this work is present an analysis of the response of the pore pressure induced by long waves of water. This phenomenon has been widely studied in the literature specialized in physical variables, being somewhat complicated to identify which are the dominant variables that significantly change the pore pressure, so the contribution of the present work is to do it in a dimensionless model to obtain dimensionless parameters that group the physical variables of the soil and the waves. The results show that long waves present greater pressure in the pore than short waves.

Resumen

En este trabajo se presenta un análisis de la respuesta de la presión de poro inducida por ondas largas de agua. Este fenómeno ha sido estudiado ampliamente en la literatura especializada en variables físicas, siendo un tanto complicado identificar cuales son las variables dominantes y que cambian significativamente la presión de poro, por lo que la contribución del presente trabajo es hacerlo de forma adimensional para obtener parámetros adimensionales que agrupan las variables físicas del suelo y del oleaje. Los resultados muestran que las ondas largas presentan una mayor presión en el poro que ondas cortas.

Parametric analysis of dynamic wave-seabed interaction		
Objetivos	Methodology	Contribution
The main objective of this study is to establish a dimensionless mathematical model to determine the dynamic response of pore pressure in terms of dimensionless parameters. These parameters include various combinations of physical wave variables such as wavelength, water depth, wave frequency, and, for the seafloor, permeability, porosity, modulus of strength, and volumetric deformation of the soil.	The methodology is based on the formulation of Biot's quasi-static equation. Starting from the characteristic values identified by Arcos, Bautista, & Mendez, 2016, the system of equations was dimensionless, resulting in dimensionless parameters that group the physical variables of the waves and seafloor. The analytical solution was compared with solutions reported in the specialized literature.	The analytical solutions reported in the specialized literature are based on an analysis of physical variables, making the mathematical study difficult to identify the most representative variables in the pore pressure response. Creating a dimensionless model allows for the creation of dimensionless parameters that involve combinations of physical variables, achieving a better physical understanding of the soil deformation affected by water waves.

Análisis paramétrico de la interacción dinámica oleaje-suelo marino		
Objetivos	Metodología	Contribución
El objetivo principal de este estudio es establecer un modelo matemático adimensional para determinar la respuesta dinámica de la presión de poro en términos de parámetros adimensionales. Los parámetros incluyen varias combinaciones de las variables físicas del oleaje como son la longitud de onda, profundidad del agua, frecuencia del oleaje y para el suelo marino corresponden la permeabilidad, porosidad, modulo de resistencia, deformación volumétrica del suelo.	La metodología se basa en la formulación de la ecuación cuasi-estática de Biot. Partiendo de los valores característicos identificados por Arcos, Bautista, & Mendez, 2016, se adimensionalizo el sistema de ecuaciones, surgiendo parámetros adimensionales que agrupan las variables físicas del oleaje y suelo marino. La solución analítica se comparó con soluciones reportadas en la literatura especializada.	Las soluciones analíticas reportadas en la literatura especializada parten de un análisis variables físicas, haciendo que el estudio matemático sea difícil para identificar las variables más representativas en la respuesta de la presión de poro. Hacer un modelo adimensional permite formar parámetros adimensionales que involucran combinaciones de las variables físicas logrando en la solución una mejor comprensión física de la deformación del suelo afectado por el oleaje.

Water waves, poro-elastic seabed, Parametric analysis

Oleaje, fondo marino poro-elástico, Análisis paramétrico

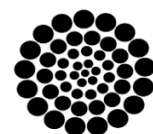
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Introduction

The phenomenon of wave interaction with the seabed has attracted the attention of maritime and geotechnical engineers in recent years. Understanding the mechanisms and processes of the wave-seabed interaction problem is particularly important for the design of foundations for maritime structures when they are subject to high-energy wave hydrodynamics. Flow conditions around structures not only affect the wave forces acting on them but can also lead to instabilities in the seabed soil generated by water waves, (Christian, Taylor, Yen, & Erali, 1974), (Lundgren, Lindhardt, & Romuld, 1989), (Smith & Gordon, 1983).

In past decades, considerable efforts have been made to study the phenomenon of wave-seabed-structure interaction. The reason for the increased interest in studying this phenomenon is that many coastal structures (such as vertical walls, marinas, oil platform columns, pipelines, breakwaters, among others) have suffered damage due to the seabed response induced by waves and not necessarily due to construction deficiencies.

Fundamentally, foundation failures of maritime structures due to seabed instabilities can be attributed to two mechanisms, which are known as liquefaction and shear failure (Silvester & Hsu, 1989). When waves propagate in the ocean, significant dynamic pressures are generated on the seabed. These pressure fields induce pore pressures and effective stresses. When the pore pressure increases and the vertical effective stresses decrease, a part of the bottom becomes unstable, leading to the generation of the liquefaction phenomenon, in which soil particles become susceptible to being transported by sea currents.

Seabed liquefaction. The seabed is considered a saturated or partially saturated porous medium (containing air bubbles) and can be cohesive or non-cohesive. From a formal perspective, the liquefaction of a non-cohesive soil is the transformation of the soil from a solid state to a liquefied state, as a consequence of the increase in pore pressure and the reduction of effective stresses. When this phenomenon occurs, the soil loses its structural strength, (Groot, Bolton, Foray, Meijers, Palmer, Sandven, Sawicki, & The, 2006), see Figure 1.

Box 1

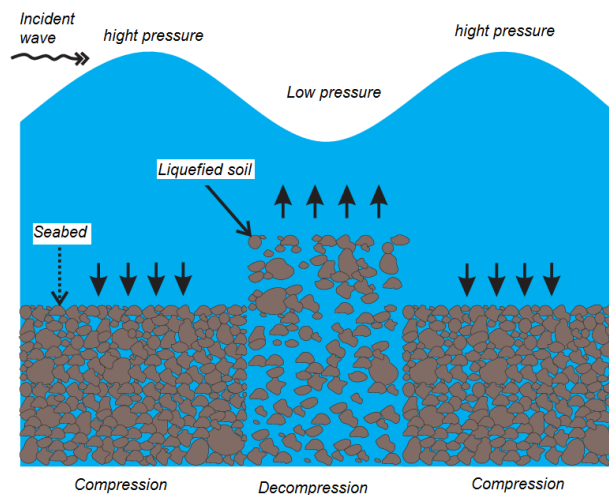


Figure 1

Description of liquefaction

Own source

The main factor causing the liquefaction phenomenon in granular media is due to the excessive increase in pore (P_p) and the reduction of shear stresses (τ) y effective stresses (σ'_z), (Marcuson, 1978). There are two mechanisms causing the accumulation of excessive pore pressure: 1) cyclic loads due to a seismic event and 2) dynamic oscillating loads from waves, (Seed & Rahman, 1978) & (Madsen, 1978).

The study of the liquefaction phenomenon is linked to the understanding of the following phenomena:

1. Soil mechanisms assuming it is dry.
2. The behavior of fully undrained, saturated soil, considering that there is no trapped air.
3. The condition of a partially saturated, undrained soil, taking into account the compressibility of the air-water mixture in the pore; this condition occurs when there are trapped air bubbles.

To determine the liquefaction potential (competition between normal effective stresses and pore pressure), the specialized literature suggests considering the influence of fluid compressibility, the solid material, and the permeability of the porous matrix. The compressibility of the water-air mixture in the pore generates a volume contraction of the mass, which causes a slight increase in pore pressure and, in turn, a slight decrease in effective stress and shear strength.

In the case of soil dilation, the compressibility of the water-air mixture in the pore causes a slight decrease in pore pressure and a slight increase in effective stress and shear strength. The loss of soil load-bearing capacity, due to the type of pressure that occurs, can generate instabilities in the foundations of maritime structures. These can be of two types: by settlement and by settlement and sliding. The orders of magnitude of displacements in real structures are on the order of meters (Groot, Kudella, Meijers, & Oumeraci, 2006).

The pioneering works in the study of the mechanisms that occur in materials subjected to cyclic loads are Biot's poro-elastic theory, (Biot, 1941), and Verruijt's equation, (Verruijt, 1969), these researchers developed three mathematical models which are: 1) Quasi-Static (QS) Model, the fluid-soil mixture in the pore is considered compressible, but the relative accelerations between the fluid and the soil are ignored.

This hypothesis leads to Biot's consolidation equation, in which the inertial terms associated with the mass of the soil and the water of the porous medium are not taken into account. 2) Partially Dynamic Approximation (u-p), the coupled equations of flow and deformation consider the acceleration of the soil mass but not the relative acceleration of the water in the pore (Zienkiewicz, Chang, & Bettes, 1980) and Full Dynamic (FD), in this case, the coupled equations of flow and deformation are formulated in such a way that they involve the relative acceleration of the soil mass and the fluid in the poro-elastic medium.

From a maritime hydraulics perspective, starting in the 1970s, various formulations began to be proposed to characterize the instability of a seabed, (Yamamoto, Sellmeijers, & Hijum, 1978) and (Madsen, 1978) used linear wave theory to study the stability of isotropic, poro-elastic granular soil. Taking into account the obliquity of the incident wave, (Tsai, 1995), proposed an analytical solution to determine the liquefaction potential of a permeable and partially saturated seabed of finite thickness. In the specialized literature, various proposals related to wave-seabed interaction can be found; these studies have treated the phenomenon as a wave train.

As the waves propagate in a shallow flow region, the phenomenon becomes highly non-linear, and the complexity of its study may require computational fluid dynamics and advanced experimental methodologies. (Jeng, Cha, Lin, & Hu, 2001) developed a numerical model to determine the effect of water pressure in the porous medium, considering the wave-seabed-breakwater mechanisms. They demonstrated that the maximum pore pressure occurs around the wave mitigation structures and depends on the wave period, water depth, and the degree of soil saturation.

In turn, (Lee & Lan, 2002) obtained an analytical solution for the propagation of periodic waves over a poro-elastic seabed of infinite depth. Considering long waves of small amplitude, (Lee, Tsai, & Jeng, 2002) determined an analytical solution to describe the seabed response, taking into account seawater seepage through a parametric study (Jeng & Cha, 2003) studied the effect of wave non-linearity and the degree of saturation on the response of the poro-elastic matrix.

Subsequently, (Liu & Jeng, 2007); (Wang, Karim, & Lin, 2007), developed parametric studies to investigate the effects of wave length and period, compressibility of the pore fluid, soil permeability, and stiffness of the deformable granular medium; the results were compared with three different criteria for momentary liquefaction. (Liu & Jeng, 2007), presented an analytical and numerical solution to describe the seepage flow induced by long waves propagating through a permeable and partially saturated seabed.

These researchers concluded that the liquefaction potential is much greater for partially saturated flows. Based on the numerical solution they proposed, they demonstrated that a flow with a higher degree of saturation induces greater wave damping. Subsequently, (Ulker, Rahman, & Jeng, 2009) compared the different poro-elastic formulations and concluded that the (QS) formulation can be used mostly in clay soils and that it does not provide significant results when wave periods are very small. These same authors show that for silty soils (fine sand) with wave periods less than 10s, the (PD) formulation is sufficient, and for sandy soils, the appropriate formulation depends on the seabed permeability and wave period.

When the soil is composed of gravel, the permeability is large, and therefore the (FD) formulation must be used. For their part, [Xiao *et al* \(2010\)](#) determined the liquefaction potential caused by the breaking of solitary waves propagating over a sloping bottom; they concluded that the maximum depth of liquefied sand depends on the wave amplitude and permeability of the saturated soil.

In parallel, [\(Jeng, Zhou, Luo, Wang, Zhang, & Gao, 2010\)](#) studied the seabed response (pore pressure, effective stress, and shear stress) affected by a combined wave-current load. The results show the effect of the sea current velocity on the response of the poro-elastic seabed. Subsequently, [\(Ye & Jeng, 2011\)](#) solved the partially dynamic u-p equations, including the shear stress at the seabed generated by the waves. The numerical results indicate the influence of the shear stress on the dynamic response of the seabed.

Using linear theory [\(Cha, Zhang, & Blumenstein, 2011\)](#) numerically obtained the liquefaction potential in a porous seabed. Other works focus on studying the seabed assuming it is formed by several layers with different physical characteristics. [\(Zhou, Xu, Wang, & Li, 2011\)](#), determined an analytical solution, taking into account a multi-layered poro-elastic soil.

They deduced that the soil characteristics (number of layers, permeability, and shear modulus) and the wave characteristics (water depth and wave steepness) have an influence on the soil response. Through an experimental model, [\(Sumer, Dixen, & Fredsoe, 2011\)](#) demonstrated that the rocks supporting submarine pipelines are stable under the effect of the hydrostatic load of very long waves but can be unstable when exposed to the movement of liquefied soil.

On the other hand, [\(Wen, Jeng, Wang, & Zhou, 2012\)](#) numerically modeled the seabed response, including the non-linear effect of waves and sea current. Their results concluded that the sea current affects the pressure distribution in the poro-elastic soil, and therefore, they suggest taking this effect into account in the study of the liquefaction potential of the seabed.

Based on the VARANS (Volume-Averaged Reynolds-Averaged Navier-Stokes) equations and the u-p approximation, [\(Zhang, Jeng, Liu, Zhang, & Zhang, 2012\)](#) developed a numerical model to study the response of granular soil affected by Bragg reflection, which is generated by a system of several breakwater structures.

Recently, [\(Zhang, Jeng, Gao, & Zhang, 2013\)](#) studied the effect of the non-linearity of waves combined with sea currents on the seabed response and determined that the pore pressure and effective stresses oscillate depending on the velocity and propagation direction of the sea currents. Other works focus their attention on the effect of seepage flow on wave deformation and the response of the porous medium. On the other hand, [\(Jianhonga, Jeng, Liu, Chan, Wang, & Zhu, 2014\)](#) numerically studied the interaction of wave breaking with a composite breakwater and its effect on the seabed. In addition, [\(Qibo, Hualing, Pandi, Shahoua, Lunlian, Linya, & Yifei, 2020\)](#) described an experimental study conducted to investigate how irregular waves affect pore pressures around a monopile on the seabed.

The monopile is a structure used in coastal engineering to support structures such as oil platforms or offshore wind farms. The researchers conducted tests in a wave flume using five different types of irregular waves. They discovered that as the wave height increased, so did the water pressure around the monopile. However, they found that the maximum water pressure inside the seabed decreases as the depth increases.

This suggests that how the water pressure is distributed changes depending on where the monopile is located and how deep it is in the seabed. These findings have important implications for the design of marine structures and coastal engineering. Understanding how irregular waves affect pore pressures can help design more resistant and safer structures, reducing the risks of damage caused by waves and improving the durability of marine facilities.

On the other hand, [\(Linya, Hualing, Pandi, Jeng, Qibo, Shaohua, Lunliang, & Yifei, 2020\)](#) described a detailed analysis of how waves and currents affect pore water pressures around a partially embedded monopile in the seabed.

Unlike previous studies that focused solely on waves, this one examines how the interaction between waves and currents influences the monopile's response. To carry out the study, experiments were conducted in wave flumes to investigate how pore water pressures vary in time and space around the monopile. The results reveal a series of significant findings. For example, when currents are superimposed on waves, it is observed that the wave profile on the side of the monopile fluctuates first, rather than in front, and the shape of the incident wave is smoothed.

In addition, they identified that the amplitudes of the pore water pressures increase and the attenuation of these pressures occurs under the influence of large waves combined with a sea current. Ponce *et al.* (2022) developed research for the implementation of an offshore wind farm off the coast of the department of Colón, Honduras, with the aim of increasing energy generation through renewable sources in the country. Furthermore, Rivas *et al.* (2025) evaluated functional connectivity in different land uses of the Conchal Reserve, part of the Baulas-Conchal Coastal Marine Biological Corridor (CBC-BC).

The theoretical works mentioned above have allowed the scientific community to have a physical understanding of the influence of the various wave parameters on the dynamic response of the soil, which is characterized by pore pressure and soil deformations. In this article, a dimensionless analytical solution is proposed where dimensionless parameters arise that group the competition of the properties of the waves with the seabed. Solving the problems using this methodology will allow for an understanding of the most significant physical variables in the phenomenon of soil instability.

Problem statement

In this work, the incident wave is considered to be a long linear wave of length λ and amplitude A_I . The water waves propagate from left to right over a porous soil of semi-infinite depth. The mean and uniform water depth is h . In the selected Cartesian coordinate system, the positive direction of x -axis is to the right, with the origin at the junction between the porous soil and the impermeable rigid soil.

The z -axis points upward, normal to the soil. In the physical model, the porous medium and the impermeable soil are identified by brown and gray regions, respectively, as shown in Figure 2. It is assumed that the porous soil is a deformable porous medium composed of a three-phase mixture. A solid phase formed by the granular medium, a liquid phase that occupies most of the porous space, and a gaseous phase that sometimes occupies a small portion of the porous space. The granular medium and the pore fluid (including both liquid and gas) can be considered together as a compressible medium.

Box 1

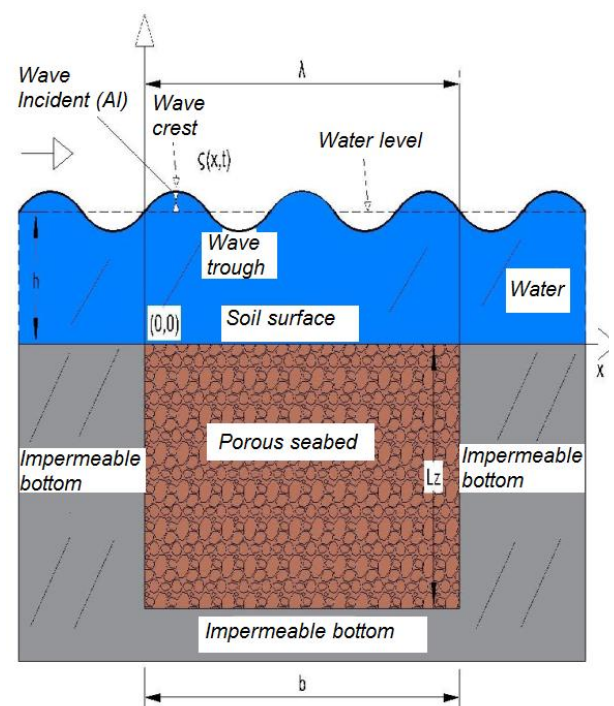


Figure 2

Profile view of the physical model under study

Own source

The physical model is divided into two different regions: 1) Wave field and 2) Poro-elastic medium. It is assumed that the seabed is composed of an assembly of fine, deformable sands, with small permeabilities and assuming that the seabed is impermeable. This condition allows for the decoupled solution of the wave field and the poro-elastic medium, from which the hydrostatic pressure at the bottom at $z = 0$ is obtained P_b in $z = 0$

Governing equations in physical variables

The present mathematical model is based on Biot's consolidation theory, following these considerations:

- Isotropic material.
- Linearity in the stress-strain relationship.
- Volumetric deformations are small.
- The water may contain trapped air.
- The water flow through the porous medium obeys Darcy's law.

Taking the above into account, the static equilibrium equations (Biot, 1941) are as follows:

$$G\nabla^2 u_s + \frac{G}{1-2\nu} \frac{\partial \varepsilon_s}{\partial x} = -\frac{\partial p_s}{\partial x} \quad [1]$$

$$G\nabla^2 w_s + \frac{G}{1-2\nu} \frac{\partial \varepsilon_s}{\partial z} = -\frac{\partial p_s}{\partial z} \quad [2]$$

$$k_s \nabla^2 P_s - \gamma_w n \beta \frac{\partial p_s}{\partial t} = \gamma_w \frac{\partial \varepsilon_s}{\partial t}. \quad [3]$$

With their respective boundary conditions: in $z = 0$

$$\sigma'_z = 0, \tau_{xz} = 0, p_s = P_b \quad [4]$$

and at $z = -L_z$

$$u_s, w_s, p_s \rightarrow 0. \quad [5]$$

Where G is the shear modulus, ν is the Poisson's ratio, u_s y w_s are soil displacements in the horizontal and vertical directions respectively. ε_s is the volumetric strain, p_s is the pore pressure, γ_w , is the specific weight of water, n is the porosity and β is the compressibility of the seabed. Equations 1-3 are the system of governing equations that will be solved along with their boundary conditions, Equation 4 and Equation 5, in this research.

Governing equations in dimensionless variables

A dimensionless mathematical model is one of the mathematical techniques used for a better appreciation of the physical variables that could be significant in the solution, and it is a contribution of the present work.

Using the characteristic variables identified by (Arcos, Bautista, & Mendez, 2016) Equations 1-3 in dimensionless variables are rewritten as:

$$\beta_0^2 \frac{\partial^2 u_s}{\partial x^2} + \beta_0 \frac{\partial^2 u_s}{\partial z^2} + \beta_0^2 \psi \frac{\partial^2 u_s}{\partial x^2} + \frac{\psi}{\Gamma} \frac{\partial^2 w_s}{\partial z^2} = -\beta_0 \frac{\partial p_s}{\partial x} \quad [6]$$

$$\beta_0^2 \frac{\partial^2 w_s}{\partial x^2} + \frac{\partial^2 w_s}{\partial z^2} + \beta_0^2 \Gamma \psi \frac{\partial}{\partial z} \frac{\partial u_s}{\partial x} + \psi \frac{\partial^2 w_s}{\partial z^2} = -\beta_0 \Gamma \frac{\partial p_s}{\partial z} \quad [7]$$

$$\beta_0 \frac{\partial^2 p_s}{\partial x^2} + \frac{\partial^2 p_s}{\partial z^2} - \beta_0 \alpha \frac{\partial p_s}{\partial \tau} = \beta_0 \Gamma \frac{\partial^2 w_s}{\partial z^2} = -\beta_0 \Gamma \frac{\partial}{\partial \tau} \frac{\partial u_s}{\partial x} + \frac{\partial}{\partial \tau} \frac{\partial w_s}{\partial z}. \quad [8]$$

and the boundary conditions are rewritten as follows, at , in $Z = 0$

$$\sigma'_z = 0, \tau_{xz} = 0, P_s = 1 \quad [9]$$

and at, $Z = -1$

$$U_s, W_s, P_s \rightarrow 0 \quad [10]$$

Where the dimensionless parameters are $\beta_0 = \left(\frac{L_z}{\lambda}\right)^2$, $\psi = (1/1-2\nu) \sim O(1)$, $\Gamma = \left(\frac{\omega \gamma_w \lambda^2}{G k_s}\right) \gg 1$, $\alpha = \left(\frac{\gamma_w n \beta \omega \lambda^2}{k_s}\right) \gg 1$. Following the orders of magnitude of the dimensionless parameters in Equations 6-8, they reduce to the following terms:

$$\frac{\partial^2 p_s}{\partial z^2} = -\frac{\partial p_s}{\partial x} \quad [11]$$

$$\frac{\partial^2 w_s}{\partial z^2} + \beta_0^2 \Gamma \psi \frac{\partial}{\partial z} \frac{\partial u_s}{\partial x} + \psi \frac{\partial^2 w_s}{\partial z^2} = -\beta_0 \Gamma \frac{\partial p_s}{\partial z} \quad [12]$$

$$\frac{\partial^2 p_s}{\partial z^2} - \beta_0 \alpha \frac{\partial p_s}{\partial \tau} = \beta_0^2 \Gamma \frac{\partial}{\partial \tau} \frac{\partial u_s}{\partial x} + \frac{\partial}{\partial \tau} \frac{\partial w_s}{\partial z}. \quad [13]$$

The u-p approximation for poro-elastic media can be reduced to a boundary value problem, assuming that the dependent variables have a harmonic oscillation with a frequency equal to that of the wave. Under these conditions, the transient governing equations of the seabed can be formulated as a linear boundary value problem. (Yamamoto, Sellmeijer, & Hijum, 1978). Following Yamamoto's model and combining Equations 11-13 the following relationship is obtained:

$$\frac{d^4 \hat{P}_s}{d\bar{z}^4} + \frac{i\beta_0[\Gamma-\psi\alpha-\alpha]}{[1+\psi]} \frac{d^2 \hat{P}_s}{d\bar{z}^2} - \frac{4i\pi^2 \beta_0^2 \Gamma}{[1+\psi]} \hat{P}_s = 0 \quad [14]$$

with its respective boundary conditions at: $\bar{Z} = 0$

$$\hat{P}_s = 1, \frac{d\hat{P}_s}{d\bar{z}} = 0 \quad [15]$$

And at $\bar{Z} = -1$ we have

$$\hat{P}_s = 0, \frac{d\hat{P}_s}{d\bar{z}} = 0. \quad [16]$$

The derivation of the analytical solution of Equation 14 is based on the exact solution developed by (Polyanin & Zaitsev, 2003), transforming into the following solution of Equation 14 as follows:

$$\hat{P}_s = c_1 e^{(-a)^{1/2} \bar{z}} + c_2 e^{-(-a)^{1/2} \bar{z}} + c_3 \cos(b^{1/2} \bar{z}) + c_4 \sin(b^{1/2} \bar{z}). \quad [17]$$

Where the parameters a and b are given by the following values:

$$a = \frac{-8\beta_0\pi^2}{(1-\mu)\left(1 + \sqrt{1 + \frac{16\pi^2\mu}{i\alpha(1-\mu)^2}}\right)} \quad [18]$$

and

$$b = \frac{i\beta_0\alpha(1-\mu)\left(1 + \sqrt{1 + \frac{16\pi^2\mu}{i\alpha(1-\mu)^2}}\right)}{2\mu} \quad [19]$$

where

$$\mu = nG\beta \left(1 + \frac{1}{1-2\nu}\right). \quad [20]$$

Using the boundary conditions from Equation 15 and Equation 16 the integration constants c_1, c_2, c_3, c_4 of term 17 are determined which in matrix form is expressed as:

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ ia^{\frac{1}{2}} & -ia^{\frac{1}{2}} & 0 & b^{\frac{1}{2}} \\ e^{-ia^{\frac{1}{2}}} & e^{ia^{\frac{1}{2}}} & \cos(-b^{\frac{1}{2}}) & \sin(-b^{\frac{1}{2}}) \\ ia^{\frac{1}{2}}e^{-ia^{\frac{1}{2}}} & ia^{\frac{1}{2}}e^{ia^{\frac{1}{2}}} & -b^{\frac{1}{2}}\sin(-b^{\frac{1}{2}}) & b^{\frac{1}{2}}\cos(-b^{\frac{1}{2}}) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad [21]$$

Next, the analysis of the results of the most representative parameters in the response of the seabed pore pressure is presented.

Results analysis

Figure 3 shows the comparison of the analytical solution in its dimensionless form (Yamamoto, Sellmeijer, & Hijum, 1978) which corresponds to Equation 22, and the present solution, Equation 17

$$P_s = \frac{\rho gh}{(\gamma_s - \gamma_w)L} e^{\left(\frac{-2\pi L}{\lambda}\right)Z}. \quad [22]$$

Box 2

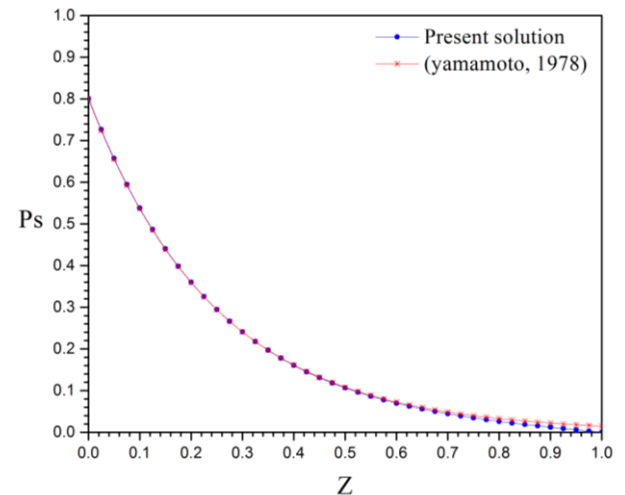


Figure 3

Comparison of the present solution with Yamamoto (1978)

Own source

Usando los valores físicos registrados en la Tabla 1, en ambas soluciones se observa que tienen muy buena aproximación, lo que significa que el modelo matemático está validado.

Box 3

Table 1

Physical values of the waves and the seabed

Water waves	
Water depth	$h=0.8\text{m}$
Wavelength	$\lambda = 1.57\text{m}$
Specific weight water	$\gamma_w = 9810\text{N/m}^3$
Seabed	
Gravitational constant	$g = 9.81\text{m/s}^2$
Specific weight of soil	$\gamma_s = 25996.5\text{N/m}^3$
Liquefaction Depth	$L_z = 1\text{m}$

Own source

The same figure shows the propagation of the pore pressure along the seabed, with its maximum value occurring at the wave-seabed interface where $Z = 0$. As the soil thickness increases, the pore pressure decreases as $Z \rightarrow 1$.

One of the main contributions of this research was to solve the mathematical model in dimensionless form to identify the dominant parameters in the pore pressure distribution. To carry out the analysis, real physical values for the waves and seabed were taken into account as shown in Table 2.

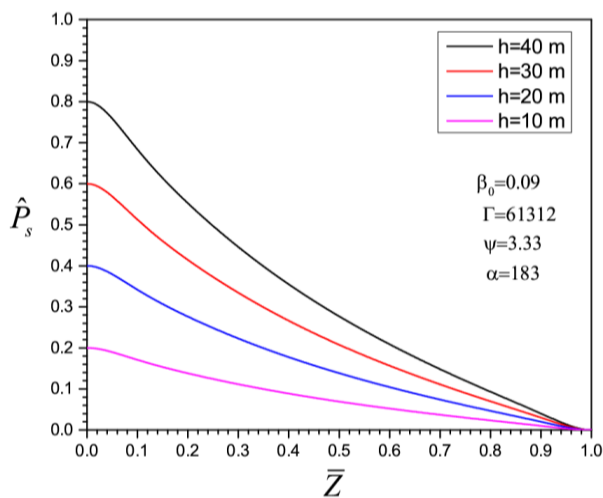
Box 4**Table 2**

Physical values of the waves and the seabed for the results analysis

Water waves	
Water depth	$h=40$ m
Wavelength	$\lambda = 100$ m
Wave period	$T = 5$ s
Seabed	
Poisson's ratio	$\nu = 0.35$
Porosity	$n = 0.3$
Degree of saturation	$S_r = 1$
Shear modulus	$G = 19999999$ Pa
Specific weight of soil	$\gamma_s = 25996.5$ N/m ³
Specific weight of water	$\gamma_w = 9810$ N/m ³
Darcy's permeability	$k_s = 0.0001$ m/s ²
Bulk modulus of water	$K_f = 1900000000$ Pa
Liquefaction depth	$L_z = 30$ m

Own source

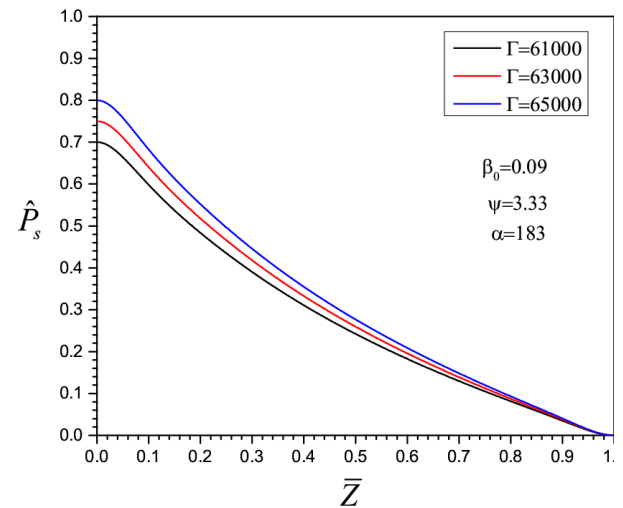
Figure 4 shows the effect of the water depth of the ocean h on the pore pressure \hat{P}_s along the bottom of the porous medium \bar{Z} .

Box 5**Figure 4**

Pore pressure distribution for different sea water depths

Own Source

The same figure shows that as the water depth increases, the pore pressure also, for a depth $h = 40$ m there is a dimensionless pore pressure value of $\hat{P}_s = 0.8$ and for values of $h = 10$ m there is a pore pressure of $\hat{P}_s = 0.2$ both at $\bar{Z} = 0$.

Box 6**Figure 5**

Pore pressure distribution for different values of the parameter Γ .

Own Source

On the other hand, Figure 5 shows the effect of the parameter $\Gamma = \omega\gamma_w\lambda^2/Gk_s$ on the behavior of the pore pressure \hat{P}_s . The results show that for increasing values of the parameter Γ , the pore pressure increases, This is because the permeability of the pore k_s is very small and there is a greater concentration of pressure or the wave is more frequent where the value of ω increases or the wavelength λ increases. Another possible cause is that the granular material over which the wave propagates has a decreasing shear strength, which is the case for loose sands.

Conclusions

In this work, an analytical solution of the dimensionless mathematical model (Equations 1-5) was determined to find the pore pressure along the seabed. The results indicate that the behavior of the pore pressure \hat{P}_s is a function of the dimensionless parameters that group the properties of the waves and the seabed. Based on the analytical result obtained, it was identified that the propagation of pore pressure is strongly affected by the following properties:

For increasing wavelengths λ which is the case for long waves, the energy contained in the waves is greater than for short waves.

When the permeability k_s is small, there is a greater accumulation of energy in the pore pressure.

For the case of an increasing frequency ω , it means that the wave train is more repetitive, causing greater fatigue on the seabed. Loose sands are more vulnerable to soil liquefaction due to the lower cohesion force between particles, which causes the shear strength to decrease.

The physical understanding of wave-seabed interaction is important to achieve safe foundations for ocean structures built on the seabed.

Conflict of interest

The authors declare no interest conflict. They have no known competing financial interests or personal relationships that could have appeared to influence the article reported in this article.

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Author contribution

Peza-Ortiz, Edebaldo. PhD: Contributed to the project idea, research method

Arcos-Hernández, Emmanuel. PhD: Contributed to the project idea, research method and technique.

García-Trinidad, Enrique. PhD: Contributed to the project idea, research method

Torres-Valle, Jose. PhD: Contributed to the project idea, research method

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