Empirical analysis of the economic sector of Mexico in R3 with fractal randomness.

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Received December 15, 2010; Accepted March 25, 2011

In this article fractal is applied to the method for is to analyze the series of time of the Economic Sectors of Mexico in relation to the IPyC. For the effect, the methodologic guidelines are applied of (Mandelbrot, 1997, p.245), (Bouchaud, 2000, p.168), (Mantenga and Stanley, 2000, p.235). It is made a statistical analysis and fractal. Previously, it is necessary to demonstrate that the behavior of this indicator has properties of similarity and affinity. With software (Fractal, 2010) the exponent of Hurst is considered, whose value is a statistical one of test which it indicates if the series of time is persistent, antipersistent or random.

Stock market, technical analysis, stock-exchange analysis, analysis fractal, theory of the chaos.

Citation: Ramos M. Empirical analysis of the economic sector of Mexico in R3 with fractal randomness. ECORFAN Journal-Mexico 2011, 2-3:169-180

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Introduction

The changes of prices of a serie of time are normally media for the increases in the prices, the logarithms performances or the absolute value of the last ones. If $P_t$ points the price of some active (price of an action as an example) in any they of negotiation, the increase in the price is defined as:

$$\Delta p(\tau) = P_t - P_{t+\tau}$$

(1)

And the relative change in the price or percentage performance $\Delta_t$ as

$$\Delta_t(\tau) = (P_t - P_{t-\tau})/P_{t-\tau}.$$  

(2)

Also, over a base of continuous composition, the performance of the price in a gave period could be calculated as the logarithm of the final price less the logarithm of the initial price:

$$\delta_t(\tau) = \ln(1 + \Delta_t) = \ln(P_t/P_{t-\tau}) = p_t - p_{t-\tau}. $$

(3)

About the absolute value of the performances, this describe the amplitude of the fluctuation, because by definition is always positive and there are not global tendencies which could be visually obvious; A key variable in the majority of the financial tools and which play a determinant role in many areas of finances that in our research is the Economy with presence of volatility in the time series of the prices.  

Daily profitability of the IPyC in the period from 03-01-10 to 03-01-11.

From the empirical point of view, it is important to model carefully any temporary variation during the volatility process. (Bouchaud, 2001, p. 11). However is normal to talk about volatility, there is not a universally accepted definition of the same. Different estimators could be used to measure the fluctuations of the prices, in particular the absolute values of the performances, the performances squared and the logarithms of the performances squared

fixing of the actives and the dynamic of the coverage strategies, as well in the determination of the price options.

2 The normal curve is focus around the average, which is present by $\mu$. The variation or dispersion around the average is express in units of the standard deviation, represent by $\sigma$. In finances, the average is a mean performance and the standard deviation is the volatility. Additionally to the average and the standard deviation, the distribution function of normal probability has to characteristics: skew and the kurtosis which also are known as third and fourth moment and future performes on their fifth moment, respectively.

$$A= \beta_1+\beta_2 (B)+\beta_3 (C)+\beta_4 (K)+\beta_5 (F)+\beta_6 \neq (T)$$

Where:

In recent studies found that some of these estimators provide practically the same empiric evidence about the long term dependence. A form to calculate the historic volatilities of the daily price registers, for different time horizons: $n,..., 3$, $2 = m$, using the next equation:

$$V_n(\tau) = (n-1)^{-1} \sum_{i=1}^{n} (P^2(\tau+i) - (P^2(\tau)))$$

Where the average value of $P^2(\tau)$ points the average time of negotiation and ($\tau$) is the time to make the transactions (including weekends and holydays of the market).

**Statistic and fractal analysis of the IPyC**

The statistical analysis of the profitability and its volatility consisted in determinate if its behavior conforms to a normal distribution and also identify if accomplish with the distribution of the heavy tails (persistence) with all the possible profiles of skews. 4.

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4 Skew topology:
Selection skew: No comparable groups because the form that the sample or facts were chose.
Information skew: No comparable groups because the form in which the facts were obtained.
Confusión skew: There is a mix of effects because a third variable.

5 Is the measurement scale which use a logarithm of the physic amount instead of the amount of itself when the facts cover a big number of values- the logarithm reduce this to a number more manageable, doing the logarithm scales for this amount of the entry especially appropriate, like that, our senses aware perceived equal frequencies.

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Daily volatility of the IPyC in the period of 03-01-10 to 03-01-11.

Source: Bolsa Mexicana de Valores, cotizaciones diarias del IPyC.

The records comprise since January 03 of 2010 to January 03 of 2011, we divide the IPyC in time horizons with a variation from ten to ten, from and interval of ten facts: 10, 20, 30... 600, 610 and 620. It calculated for each horizon the kurtosis. The average of this statistical is calculate in scale log-log.
Normality Test

It is opportune emphasize that in statistics is possible to demonstrate that if we consider a sample of size N belonging to a population that normally distribute (with measurement \( \mu \) and standard deviation \( \sigma \)) named sample will have a normal distribution of measurement \( \bar{x} \) and standard deviation.

\[
\frac{\sigma}{\sqrt{n}}.
\]

The theorem of the central limit establishes when the simple of size \( N \) is big enough, the distribution of the simple is approximately normal.\(^6\)

Additionally, to the measurement and to the standard deviation, the function of distribution of normal probability has two characteristics: skew and the kurtosis, which are also known as third and fourth moment, respectively.

The skew is an indicator which measure the curve symmetry. In the case of a normal perfect curve, the skew will be equal to cero.

If this is negative, the curve will be biased to the left; if this is positive the curve will be biased to the right.

\[
Sesgo = \frac{\sum(x_i - \mu)^3}{(n-1)\sigma^{3/2}}.
\] (5)

Where:

\( x_i \) = Level of IPyC on each period expressed in days
\( \mu \) = Average in the period
\( \sigma \) = Standard deviation

The kurtosis is the indicator which measures the lifting of the curve respect to the horizontal.

This situation is presented when there are many observations far from the average. To this phenomenon of high kurtosis is also known as fat tails. The kurtosis of a perfect distribution is equal to 3.

\[
Kurtosis = \frac{\sum(x_i - \mu)^4}{(n-1)\sigma^4}
\] (6)

Therefore, we present the obtained results in order to know the skew and the kurtosis of the IPyC in the Chart 1.
Skew and Kurtosis del IPyC Vs Economic sectors in Mexico

<table>
<thead>
<tr>
<th>Sector</th>
<th>Mean</th>
<th>Std. Dev</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>t Error</th>
<th>t Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>AXIMOF</td>
<td>3.101</td>
<td>7.249</td>
<td>51.888</td>
<td>6.35</td>
<td>1.69</td>
<td>148</td>
<td>0.019</td>
</tr>
<tr>
<td>INMO</td>
<td>7.123</td>
<td>5.374</td>
<td>21.169</td>
<td>56.820</td>
<td>6.54</td>
<td>148</td>
<td>0.019</td>
</tr>
<tr>
<td>OLMEN</td>
<td>6.473</td>
<td>2.293</td>
<td>0.398</td>
<td>38.238</td>
<td>0.25</td>
<td>148</td>
<td>0.182</td>
</tr>
<tr>
<td>XTRACTI</td>
<td>3.176</td>
<td>6.416</td>
<td>3.22079</td>
<td>41.530</td>
<td>2.07</td>
<td>148</td>
<td>0.221</td>
</tr>
<tr>
<td>RANSFOR</td>
<td>3.733</td>
<td>8.866</td>
<td>6.2169</td>
<td>3.2881</td>
<td>0.17</td>
<td>148</td>
<td>0.120</td>
</tr>
<tr>
<td>ONSTRUO</td>
<td>1.767</td>
<td>0.072</td>
<td>0.0375</td>
<td>4.682</td>
<td>0.02</td>
<td>148</td>
<td>0.496</td>
</tr>
<tr>
<td>OMERCIO</td>
<td>1.701</td>
<td>9.699</td>
<td>5.7576</td>
<td>39.260</td>
<td>1.81</td>
<td>148</td>
<td>0.148</td>
</tr>
<tr>
<td>OMTRACO</td>
<td>4.860</td>
<td>8.445</td>
<td>3.87087</td>
<td>92.540</td>
<td>20.00</td>
<td>148</td>
<td>0.466</td>
</tr>
<tr>
<td>SERVICIO</td>
<td>2.324</td>
<td>0.125</td>
<td>0.01656</td>
<td>36.775</td>
<td>0.62</td>
<td>148</td>
<td>0.396</td>
</tr>
<tr>
<td>SERVICIO</td>
<td>1.348</td>
<td>1.501</td>
<td>1.1241</td>
<td>1.124</td>
<td>0.11</td>
<td>148</td>
<td>0.542</td>
</tr>
<tr>
<td>CIERRE</td>
<td>7.412</td>
<td>3.520</td>
<td>12.0871</td>
<td>44.690</td>
<td>2.64</td>
<td>148</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Source: Own elaboration with Software SPSS 17.0.

Considering the Average of the Extractive Sector for be the major participation with 47.17% in IPyC as principal detonator of the Commerce Sector’s activities which is the most affected by: little participation of 11.47%.

It is calculated the statistical test:

\[
LM = \frac{N \cdot (\text{skew} - 3)^2}{6} + \frac{N \cdot (\text{kurtosis} - 3)^2}{24}
\]

(7)

Where \( LM \) is a test statistic and is distribute according to a bi-squared with two freedom degrees. The hypothesis is considered null the value 0.5 (value which correspond to the normal or gaussiana curve) with a 95% of the confidence level.
ECORFAN Journal-Mexico
April 2011 Vol.2 No.3 169-180

The probabilistic analysis consisted in determine which distribution of the probability better adjusted to the historic behavior of the IPyC, using the SPCC software. Once identify the statistic distributions, it proceeded to analyze its parameters. The purpose of this analysis was to find distribution of fat tails (potency laws behavior).

Fractal analysis

The fractal analysis consisted in detect if the tail of probability distribution of the profitability and the volatility accomplish with the law of potency and, also, if the time series of the IPyC has properties of self-similarity and self-affinity through the stimiation of the Hurst exponent (H).

In first place, are study the distributions of the potency law for its characteristic of being self-similarly in different scales or exponents; in second place, the Hurst’s exponent is estimated using the Fractal software; finally, are studied the self-relation functions.

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7 The SPSS software was used for a better adjust of the probability distribution of the behavior of IPyC. This software is developing to analyze situation sensible to the risk, order the probability distributions, starting with those that better adjust the facts.

8 Harold Edwin Hurst, Design the Assuan dam (Egypt) and studied temporal series related with the caudal of the Nilo River and the problems of water storage. Used a facts base of 800 years of archives and notices that were a tendency of a year of high caudal followed by other of higher caudal, and for one of low caudal were followed for one lower; with this motive, made a new statistical method (R/S).

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The oldest and famous potency law in the economy is the wealth distribution of Pareto (Bouchaud J, 2002, p.67). The individual wealth distribution $F(X)$ is frequently described, on it asymptotic tail, for a potency law:

$$F (X) \cong \frac{X^\mu}{X_0^{1+\mu}}, X \gg X_0$$

(8)

Where:
- $F(X)$ is the wealth distribution of an economy.
- $\mu$ is characteristic of the parameter of the growing of big wealths ($X$’s).
- $X$ wealth of the economic agents.

Conform the value of $\mu$ is smaller that 1, the growing is slower, and the gap between the richest and the poor is bigger. According to Pareto, in a population of $N$ size, the quotient of the biggest wealth and the typical wealth (average) grown as $N^{1/\mu}$. In the case of $\mu < 1$, the average wealth diverges: this corresponds to an economy in which a finite fraction of the total wealth is in the hands of few people. In the other hand when $\mu > 1$, the richest people only have a fraction of the total wealth (in the limit when $N \to 1$). Empirically, the exponent is in the rank $1 \leq \mu \leq 2$. This exponent of Pareto also describes the entry distribution, the companies’ size, the pension funds, etc.. (Bouchaud, 2001, p.123).

Where $R$ indicates the Rank (for example the difference between the maximum cumulative download of the river and the mine, during the period of study) and $S$ the typical deviation of the observed values of the $X$ downloads.

The evaluation of the Hurst’s exponent is the first step in the recognition and characterization of the complex dynamic in series of time. This analysis allows difference a random series of other not eventful and helps in the qualitative description of financial markets behavior.

In the other hand, a series of time that have some level of predictability will show positive self-correlation. Otherwise a series with negative self-correlation does not have predictability level. An exponent of Hurst in the rank $0.5 < H < 1$ correspond to temporal series which sow persistence (a period of growing is follow by an analogue one). This means that there is more possibility that an increase will be followed by a similar one. It has positive self-correlation. While the values placed in $0 < H < 0.5$ correspond to a behavior anti-persistent (a period of growing is follow by a growing one or vice versa), there are more probabilities that the next period will be under the average. It has negative self-correlation.

Finally, if $H$ is equal to 0.5 correspond to a random movement; an increase could be followed by a low or by other similar (the movements do not unfold any memory). It has self-correlation equal to cero.

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9 An exponent with rank $0.5 < H < 1$ corresponds to temporal series which sow persistence (a growing period is followed by an analogue one). We present easier form of limit shortening of finite to infinitesimal movement:

$$\lim_{l \to l_0} E^{a_{l_{i-j}}} = \left[ \frac{G(h)}{1 - J} \right] + J^i + \frac{b}{J} \left[ \lim_{l \to l_0} \frac{E^{a_{l_{i-j}}} + J^i}{J} \right] + \theta E^{n_{l_{i-j}}} = \lim_{l \to l_0} \frac{G(h)^{1 - J}}{1 - J}$$
The exponent of Hurst “H” is bigger than 0.5 and minor that 1, which means that the IP^YC has a persistent behavior, the daily information has a fractal behavior, because the exponent H is bigger than 0.5 and minor than 1. In other words, the most probable is that continuous with the higher tendency in the long term, existing under noise in the analyzed facts.

And to detect the existence of memory in volatility time series of the price X (τ), were use.

\[ C(\Delta \tau) = \frac{\langle X(\tau + \Delta \tau)X(\tau) \rangle}{\langle X^2(\tau) \rangle} \]  \hspace{1cm} (9)

The form in which the correlation was quantified was through the determination of the Hurst exponent, H. The hoped relation between the value of a time series t and its values on time \( t + \tau \) is a measure of the present correlation in a series.

A stationary \(^{10}\) time series has a correlation which only depends of the time period \( \tau \) between the two observations and the growing until cero, the faster enough to \( \tau \) increase, reflecting the fact that the influence of the former values reduce with considerable intervals. The velocity of this decreasing is a measure of the “memory” of the stochastic process.

Since the time series are conformed by discreet facts, \( \{X_k\}_{0 \leq k < N} \), such that \( X_k = X(k\tau_0) \), where \( \tau_0 \) is the minimum interval of time, the self-correlation function is define as:

\[ C(n) = \frac{\text{Cov}(n)}{\text{Cov}(0)} \]  \hspace{1cm} (10)

Where

\[ \text{Cov}(n) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N/2} X_kX_{k+n} \]  \hspace{1cm} (11)

\[ \text{Cov}(0) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N/2} X_k^2 \]  \hspace{1cm} (12)

N represents the total number of facts. The behavior of the self-correlation functions when \( 0 \to \tau (0 \to N) \to \infty \) determinate the local properties of the time series.

For a white noise where the value in an instant is no correlated with a previous value, the function of correlation is \( C(\tau) = 0 \) for \( \tau > 0 \).

Many of the no stationary time series are characterized by correlation of short term with a scale of time characteristic, \( \tau_0 \), and a function of exponencialmente decreasing self-correlation for example:

\[ C(\tau) \propto \exp(-\tau/\tau_0) \]  \hspace{1cm} (13)

If the correlation function \( C(n) \) climbs with the interval \( n \) like:

\[ C(n) \propto n^{-\beta} \]  \hspace{1cm} (14)

For \( n \) too big where \( 0 < \beta < 1 \), so \( \{X_i\} \) is called correlation to long term, process with memory to long term. The reason to use this terminus is that \( C(n) \) reduce slowly, in such way that \( \sum_{n=1}^{N} C(n) \) diverge when \( \infty \to N \).

---

\(^{10}\) Demonstrating with the principle of Economic Seasonality:

\[ Gh \left[ \frac{\partial dJ_l}{\partial t} \right]^{1/2} = Gh \left[ \frac{dJ_l}{dt} \right]^{1/2} + Et \]

ISSN-Print: 2007-1582- ISSN-On line: 2007-3682

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\(^{11}\) Being \( \sum a_k \) A series which character is desirable to establish and being \( \sum_{k=1}^{\infty} a_k \) a convergent series with plus \( U \), verifying that \( a_k \leq a_k \), so \( \sum a_k \) converge and its sum \( S \) is minor or equal to the sum \( U \).The series \( \sum_{k=1}^{\infty} a_k \) The series is a majoring series of the given series.

To detect the existence of memory in the time series of price volatility, $X(\tau)$, was used the function of correlation.

$$C(\Delta \tau) = \langle X(\tau + \Delta \tau)X(\tau) \rangle/\langle X^2(\tau) \rangle$$  \hspace{1cm} (15)

Scaling of the Hurst exponent in Fractal Methods of R3.

The statistic method of rescale rank (R/S) used by Mandelbrot and Wallis, is based in the previous analysis of Hurst. It allows the calculation of the self-similarity parameter $H$ to measure the intensity of dependence of long term in a series of time. For time series of length $n$

$$X = \{X_\tau; \tau = 1, 2, ... n\}$$  \hspace{1cm} (16)

$R/S$ is define as the quotient of the maximum normalized route of the integrate signal $R(n)$ between standard deviation $S(n)$:

$$R(n) = \max_{0, r_\tau: r = 1, 2, ..., n}\{0, r_\tau: r = 1, 2, ..., n\} - \min_{0, r_\tau: r = 1, 2, ..., n}\{0, r_\tau: r = 1, 2, ..., n\}/\sqrt{S^2(n)}$$  \hspace{1cm} (17)

Where:

$$\frac{\max(...)-\min(...)}{2}$$  \hspace{1cm} (18)

It is the values route

$$r_k = \sum_{r=1}^{k} X_r - \frac{k}{n} \sum_{r=1}^{n} X_r$$  \hspace{1cm} (19)

It is the maximun value less the minimun

$$S(n) = \left[ \frac{1}{n} \sum_{r=1}^{n} \left( X_r - \frac{1}{n} \sum_{r=1}^{n} X_r \right)^2 \right]^{1/2}$$  \hspace{1cm} (20)

Standard deviation

Scaling of the Hurst exponent in the Fractal Methods of R3.
A trustable measurement of $S(n)$ requires of a fact of samples with a constant interval because the wonder difference between the constant values of $X$ is a function of the distance which separate them. The exactness in the determination of $H$ depends of the number of facts used in the calculation. If named number is reasonable big, it hopes that the $R=S$ give information about the self-similarity of all the time intervals of the Economic Sectors in Mexico in R3 with fractal randomness, depending in the Skew of the Operation in the IPyC.

**Conclusions**

After have applied the analysis in base of the R3 and the Fractal Randomness with Evidence for the Economic Sectors of Mexico we get 3 meaningful results:

The fractal analysis of the IPyC allows the determination in an suitable form of the market movements that’s why we determinate ranks with higher levels of confidence, the prognostic will be more exact and on this form the real higher tendency or lower, in other words nominal terms in order to carry them to logarithm level through the Chartism with 7 Economic Sector of Mexico (Extraction, transformation, building, commerce, communications and transport, service and various).

In correlation with the IPyC f the Mexican Stock was present framed in 130 companies (all of them stock), its total lost was of 0.2% of 100 in 4 companies (AGRIEXP, CNCI, QUMMA and TEKCHEM) and was then only one with wrong treatment of information with technic analysis and represents a category of self-similarity.

Because in this stock is exigency that the Chartism looks identical to different scales, this is the proof of a good management and selection of numeric or statistic facts that preserve with the scale change (from nominal to real), and it carry us to focus to the quasi self-similarity that required a Chartism approximately identical to different scales.

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12 Agro Industrial Exportadora, S.A. de C.V. (AGRIEXP), now a days is a company that act as pure controller, which through of its subsidiaries companies, process fruits preparations for industries of yogurts, ice creams and confectionery.

13 University CNCI is a company that Works to offer services of education to the Mexican market.

14 Qumma started its quotation in the Mexican value market in June 29th of 1994, in those days under the denomination of Fernandez Editors, S.A. of S.V. and with key of quotation Gfesa. Since that date until December 31th of 1998, its structure was vertical, in other words Gfesa controller and Fesa as direct subsidiary.

15 It is a company with more than 50 years of experience in the multimodal transport field. Offer integral services of logistic, land and sea ports supported by a solid operative, technologic structure and capable human resources.
It is a suitable tool of prediction of the moments in which will happened the important events related with the evolution of the market which allow us study to detail the stochastic noise of sample of the IPyC with the Economic Sectors and all the explosion located of the volatility could be easily identify. This characteristic, known as volatility clustering, invokes intermittent fluctuations similarly in turbulent flows.

This effect could be analyzed more quantitatively: the temporal correlation function of the daily volatility could be adjusted by an inverse potency of the displacement, with small exponent in the rank 0.1-0.3.

This slow decreasing of the correlation function of the volatility drives to a multifractal behavior of the Price changes: the kurtosis of the differences of price logarithms only decrease as small potency of the time, instead of the inverse of the time as would be the case id the volatility were constant or if it had correlation in the short term.

That sallow decreasing of the kurtosis has important consequences in the rank of prices theory and according to the negotiated volume because they are strongly correlated.

On each transaction there is a probability that the price change, and after a certain horizon of time, exist a total change of the price. We obtain the price change (because the cumulative distribution obeys to a cubic invers law, the distribution function of probability by differentiation) and obey to a quartic invers law (of fourth moment).

Should be taking on account the periodicity of the counting, the “cycle rank” in which we are. Makin it we will avoid disgusting and expensive errors in the moment to made prognostic of the prices with the inclusion of the time as variable (our research consider 1 fiscal year), this means that there is not a characteristic scale for the diffusion of prices because it is being define around a media that by itself is changing (like the economic universe in which we live), so the laws of diffusion change and in particular they adopt a form of free scale or Fractal randomness in evidence of the Economic Sectors of Mexico.

References


