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Comparison of European and Asian valuation of options with underlying average and stochastic interest rate by Monte Carlo simulation

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This paper proposes a methodology to obtain the price of an asian option with underlying averagethrough Monte Carlo simulation. It is assumed that the interest rate is driven by a mean reversion process of Vasicek and CIR type with parameters calibrated by maximum likelihood. The simulation includes the quadratic resampling which reduces the use of computational resources, in particular the method improves the generation of variance covariance matrix. The proposed methodology is applied in the valuation of options with underlying price AMXL. The results show that by comparing prices of european options, with both simulated and published by MexDer with their asian counterparts, asian options prices are lower in the case of call and put options in the money. For put options simulated prices were lower in all cases. It was also found that the difference increases as the time to maturity of the option increases.

Geometric Avarage, Matematic Modelation, Asian Options

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Introduction

The Asian[17] options, also known as average options are an important class of instruments in exotic derivatives, they belong to the dependent methods of its trajectory, the value of the option on maturity depends not only on the value that reaches the active underlying on maturity, but also of the evolution that has it throughout the life of the contract. Similarly, Asian options can be European or American. There is an extense variety of underlying in these kind of contracts: currencies, equities, interest rates, goods (commodities), insurance and energy (electricity).There are multiple reasons why they are popular and some of which are mentioned in the development of this research.

The purpose of traiding Asian options is to reduce price volatility of the underlying asset just before the due date[18]. These are useful when is made frequent transactions on the same asset in a given time. It is cheaper to buy an Asian option to consider *n* different prices for the same asset (through averaging) on maturity, to buy *n* options of the same asset on different maturities, which consider different *n* premiums being very costly in both alternative which risk coverage is very similar.

Asian options are popular in the financial industry because their premium is less than the European one and they are less sensitive to variation in prices of the underlying assets.

The averages commonly used in the option contracts of this type are the arithmetic or geometric of the underlying.

Most of the Asian options are trade with discrete sampling, if is sampled with daily prices this can be approximated by continuous sampling.

Asian options are classified in various ways. For example, if the strike price depends on a fixed amount, the option is known as an Asian option with fixed strike price or with average price. If the exercise price is proportional to the price of the asset, then it is an Asian option with strike price or an option with floating average strike price. Another distinction can be made according to the nature of the mean or average, if is arithmetic or geometric, both with different weights in the previous observations. The average can be calculated with discrete sampling, with finite number of previous embodiments or by continuous sampling. In practice, all contracts are written by arithmetic mean with discrete sampling, although many papers in the literature consider the continuous case.

Some of the reasons justifying the negotiation of Asian options are the following; because the contracts only depend on the final price of the underlying that are more vulnerable to sudden shocks or large price manipulation, Asian options are less sensitive to these phenomena. Some agents prefer Asian options as hedging instruments, as they may be exposed to the performance of the underlying on a timespan. In addition, Asian options are cheaper and easier to cover than their plain vanilla counterparts.

[17] These options are called Asian because they were operated in some Asian markets to discourage the over year to maturity are first used in 1987, when the office of Bankers Trust in Tokyo's used them to value options with average price on contracts of a barrel of oil .

[18]A more detailed description can be found in Haug (2006).

This last results if you take into account that the average volatility will usually be less than the underlying one, the closer to the due date is, the lower the average uncertainty will be. This means that there is less dependence on the Asian option price and changes for a plain vanilla option with the same maturity

With respect to the valuation of these options under probabilistano approach there are solutions for Asian options with arithmetic mean. This approach assumes that the core follows a geometric Brownian motion which is equivalent to assume that the core follows a lognormal distribution. Unlike the geometric mean modeled as a product with correlated lognormal random variables, the arithmetic mean is the sum of the correlated lognormal random variables, which is why there is not a closed form of expression for distribution function of this amount, see Linetsky (2004). The same problem results in the valuation of a basket option, whose price depends on the arithmetic mean of several assets.

The price of an Asian option with arithmetic mean can be approximated by its geometric counterpart in several ways. For example, Turnbull and Wakeman (1991) approximate the price of an option with matching arithmetic to geometric moments with media partners. It can be use the Monte Carlo methods with variance reduction, with the price of the geometric choice and the control variable to calculate the price of arithmetics, see Glasserman (2003). An approximation formula of Asian options shown in Levy (1992).

In literature exists solutions in a closed form for Asian options with geometric mean with continuous sampling, see for example, Angus (1999) Vorst (1992) Kwok and Wong (2000). Dai (2003) also proposes a binomial model to value options of European and American geometric mean. These models have the advantage that the geometric mean of lognormal random variables results to have a lognormal distribution. Once you have the joint density function of the underlying price and the average, the price of the option is given by the expectation of the function of the payment under a neutral measure of risk

In Fouque and Han (2003) combine the approaches of Fouque, Papanicolaou, Sircar (2000) and Vecer (2002) to valuate Asian options under the hypothesis of reversion to the average volatility. His work aims to calibrate parameters with market prices of European options and the price of the option with a numerical algorithm, which consists of solving two partial differential equations with coefficients dependented on time. However, the accuracy of this method was not shown because it does not provide an analytical solution for the option price.

On the other hand, there are several studies that relax the assumption of constant interest rate model of Black and Scholes modeled as a stochastic process. The first work that incorporates stochastic interest rate option valuation is due to Merton (1973).

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In Goldstein and Zapatero (1996) derive a formula for valuing an option on a share with the assumption that the interest rate is driven by an endogenous process of Vasicek type in an equilibrium approached to the interest rate. Kimy Kunitomo (1999) extends the model of Black and Scholes (1973) to modify the formula with deterministic interest rates and adjustment terms driven by the volatility of the interest rate. Recently Kim, J.-H., Yoon, J.-H. and Yu, S.-H. (2013) propose a model of option pricing in which the interest rate is driven by a process of Hull and White type with a focus on stochastic volatility in order to evaluate the sensitivity of the option price that changes in the interest rate.

In this way in literature there is a wide variety of methods to determine the price of an Asian option with arithmetic mean. In general terms, there are methods based on the solution to a partial differential equation, analytical approaches, lower and upper bounds, binomial trees, processing methods and methods of working of Monte Carlo.This work does not pretend to provide a comprehensive view of the above methods, in other words the objective is to contribute with a methodology based on the Monte Carlo simulation

In this research the price of an Asian option with average strike price on the arithmetic mean is obtained by Monte Carlo simulation. The assumptions of the proposed methodology are: the interest rate is stochastic and is driven by a process of mean reversion, specifically, it is assumed that the dynamics of the interest rate is modeled by a process of Vasicek and CIR types.

To obtain the price of the option under the previous assumptions using Monte Carlo simulation methodology with the quadratic resampling of Barraquand (1995), which improves the accuracy of the calculations and reduces the use of computational resources.

This paper is organized as follows: in the next section payment functions of an Asian option considering the arithmetic and geometric averages also compare with the option prices obtained with the model of Kema, Vorst (1990) Black and Scholes (1973); during the section 3 squares method of Barraquand resampling (1995) ; Section 4 presents the methodology for determining the price of an Asian option with underlying average with stochastic interest rate process driven by a reversion to the mean, in section 5 a comparative analysis is made between prices of European and Asian options with underlying score obtained with the proposed methodology; Finally, conclusions are presented.

Asian options with arithmetic and geometric mean

An option with average strike price is an Asian option where its payment depends on an strike price equal to the arithmetic average of the price of the underlying price during the life of the option. There are several ways to generate average values of the underlying price S_t. If the behavior of S_t is observed in a discrete time intervals t_i in a equidistant way during interval time $h := T/n$, then a serie of prices $S_{t_1}, S_{t_2}, \ldots, S_{t_n}$ is obtained. For example, for the aritmethic average:

$$\frac{1}{n}\sum_{i=1}^{n} S_{t_i} = \frac{1}{T} h \sum_{i=1}^{n} S_{t_i}.$$

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If the observations are performed continuously during an interval time $0 \le t \le T$, the previous average corresponds to the integral:

$$\hat{S}_a := \frac{1}{T}\int_0^T S_t dt, \tag{1}$$

Most of the contracts are Asian options on arithmetic mean. In other cases, the geometric mean, which is used can be expressed as:

$$\left(\prod_{i=n}^{n} S_{t_i}\right)^{1/n} = \exp\left(\frac{1}{n}\ln\prod_{i=n}^{n} S_{t_i}\right) = \exp\left(\frac{1}{n}\sum_{i=1}^{n}\ln S_{t_i}\right).$$

Therefore the geometric mean with continuous sampling rate is the integral:

$$\hat{S}_g := \exp\left(\frac{1}{T}\int_0^T \ln S_t dt\right). \tag{2}$$

If the averages $\hat{S}_a$ and $\hat{S}_g$ are built in the period of time $0 \le t \le T$, then corresponds to a European option. If the early exercise is allowed in $t < T$, and it is rewittren to $\hat{S}_a$ and $\hat{S}_g$, for example:

$$\hat{S} := \frac{1}{t}\int_0^t S_\theta d\theta.$$

With a given average $\hat{S}$ given with the arithmetic mean of (1) average or geometric mean value in (2) payment functions of Asian options are summarized as shown in the following table:

Summary table of payment functions of an Asian option

Pay function	Name of the option
$\max(\hat{S}-K,0)$	Option with underlying average
$\max(K-\hat{S},0)$	Option with underlying average
$\max(S_T-\hat{S},0)$	Option with average strike price
$\max(\hat{S}-S_T,0)$	Option with average strike price

Table 1

Source: Prepared

By comparing the features of a previous payment with an option of the Vanilla Plan, for an Asian option with underlying average $\hat{S}$ has to replaced S, while for an Asian option with average strike price $\hat{S}$ substitutes K.

A notable feature of Asian options is to use the fact that the underlying average has lower volatility just before the due date.[19]

Kemma and Vorst formula to approximate the price of an Asian option with a continues geometric mean

Kemna and Vorst (1990) show that a continues geometric average of Asian option can be

[19]A more detailed description of the advantages of the use of averages is in Wilmott (2006).

valued with the same approach of an option of a Vanilla plan, It should be change the parameters of volatility, σ by σ_a, and the cost of hauling, b by b_A.

According to Kemna and Vorst the formulas for valuing the call and put options are:

$$c \approx S_t e^{(b_A - r)(T-t)} \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2), \quad (18)$$

$$p \approx K e^{-r(T-t)} \Phi(-d_2) - S_t e^{(b_A - r)(T-t)} \Phi(-d_1).$$

Where d_1 and d_2 are given by:

$$d_1 = \frac{\ln(S_t/K) + (b_A + 1/2\sigma_a^2)(T-t)}{\sigma\sqrt{T-t}}, \quad (19)$$

$$d_2 = d_1 - \sigma_a \sqrt{T-t}.$$

The function $\Phi(d)$ is the cumulative distribution function of $\mathrm{E} \sim \mathrm{N}(0,1)$. The adjusted volatility is equal to:

The adjusted volatility is equal to:

$$\sigma_a = \frac{\sigma}{\sqrt{3}},$$

While the cost of carry is set:

$$b_A = \frac{1}{2}\left(v - \frac{\sigma^2}{6}\right).$$

As mentioned in the introduction the price of an Asian option is less than the European one with the same parameters, so we proceeded to investigate this claim empirically, to achieve this goal, considering the following parameters:

Parameters used to calculate prices of Asian options buying and selling with the Kemna and Vorst (1990) and models of Black and Scholes (1973).

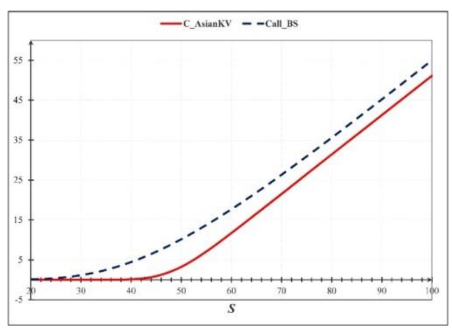
S_t	K	t	T	$\tau = T - t$	r	σ
50	50	0	1	1	0.1	0.4

Table 2

Prices of the options of buying and selling according to the parameters in Table 2 are calculated. Results for purchase options are shown in Graphic 1, the price of the underlying is considered from $S_t = 20$ to $S_t = 100$ in increments of 5.

It is observed that out of the money options for the price of an European call option is greater than the Asian one, for options inside the money price of a European call option is greater than the Asian one with the same parameters , the same is observed for the money options. Similarly with put options are calculated the prices, the results are shown in Graphic 2

Comparing prices of Asian call options obtained with the Kemna and Vorst (1990) and the Black and Scholes Model (1973)



Graphic 1

Source: Own elaboration

Comparing prices of Asian call options obtained with the Kemna and Vorst (1990) and the Black and Scholes Model (1973)

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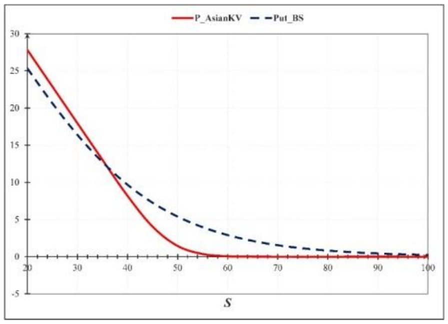

Graphic 2

Source: Own elaboration

According to the results obtained for put options in Graphic 2 is shown that for options out of the money, within money and inside the money the price of an Asian option is lower than the European one, in all cases. However, for options deeper in money the price is more in Asian than the European.

Quadratic resampling

One of the most representative items of computational finance is the determining of the price of an asset that does not have a closed formula, this methodology generally consists of calculating the expectation of a function of a payment, for example the payment function of an option.

Monte Carlo[20] simulation is a widely tool used in practice, the basic idea is to estimate the hope of functions of random variables using the average of a large number of samples obtained from these simulations, although for proper accuracy is required to increase the number of simulations, with current computing resources where point is not a problem. However, in complex situations is required to simulate a process with several variables, the computations become an issue that deserves special attention.

[20] Véase Boyle (1997) et al respecto al uso de la simulación Monte Carlo en finanzas computacionales.

In this context Barraquand (1995) notes that when random variables are generated, statistics show that in general do not coincide with the theoretical statistics. To take advantage of the properties of the theoretical formulas containing such statistics, it is necessary to transform the data in order to establish equality between the parameters of the sample and the theoretical parameters.

Definitely the key parameter is the variance-covariance matrix of a random vector.

Following this technique is described:

It is $\underline{X} = \left(X_1, \ldots, X_n\right)^{\bullet}$ a random vector of n dimension average:

$$\underline{m}_{\underline{X}} = E\left[\underline{X}\right] = \left(E[X_1], \ldots, E[X_n]\right)^{\bullet}, \tag{3}$$

And variance-covariance matrix:

$$\Sigma_{\underline{X}} = E\left[\left(\underline{X} - \underline{m}_{\underline{X}}\right)\left(\underline{X} - \underline{m}_{\underline{X}}\right)^{\bullet}\right] \tag{4}$$

$$= E[\underline{X}\,\underline{X}^{\bullet}] - \underline{m}_{\underline{X}}\underline{m}_{\underline{X}}^{\bullet}.$$

To estimate $\underline{m}_{\underline{X}}$ and $\Sigma_{\underline{X}}$, M simulations are run and then calculate the sample statistics such as:

$$\hat{\underline{m}}_{\underline{X}} = \frac{1}{M}\sum_{k=1}^{M}\underline{X}^{k} \tag{5}$$

And

$$\Sigma_{\underline{X}} = \frac{1}{M}\sum_{k=1}^{M}\left(\underline{X}^{k} - \hat{\underline{m}}_{\underline{X}}\right)\left(\underline{X}^{k} - \hat{\underline{m}}_{\underline{X}}\right)^{\bullet}, \tag{6}$$

Where $\underline{X}^{k}$ is the vector of the k-th simulation. By developing the expression of the sample variance is:

$$\Sigma_{\underline{X}} = \frac{1}{M}\sum_{k=1}^{M}\underline{X}^{k}(\underline{X}^{k})^{\bullet} - \hat{\underline{m}}_{\underline{X}}\hat{\underline{m}}_{\underline{X}}^{\bullet} \tag{7}$$

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According to the law of large numbers when M is too big, $\hat{\underline{m}}_{\underline{X}}$ and $\hat{\Sigma}_{\underline{X}}$ are closer to $\underline{m}_{\underline{X}}$ and $\Sigma_{\underline{X}}$ with good precision.

If M is small the precision is not significant. However, it is posible to modify the data in $\underline{X}$ such that the sample mean and variance-covariance matrix matches the mean of the variance-covariance theory.

In making the estimates based on data $\hat{\Sigma}_{\underline{X}}$ is an square matrix defined as symmetric and positive which the square root of the matrix $\hat{\Sigma}_{\underline{X}}$ exist and is regular:

Consider the matrix:

$$H = \sqrt{\Sigma_{\underline{X}}}\left(\sqrt{\Sigma_{\underline{X}}}\right)^{-1} \tag{8}$$

And the vector:

$$\underline{Y} = H\left(\underline{X} - \hat{\underline{m}}_{\underline{X}}\right) + \underline{m}_{\underline{X}}. \tag{9}$$

If is executed M simulations the new vector is:

$$\underline{Y}^k = H\left(\underline{X}^k - \hat{\underline{m}}_{\underline{X}}\right) + \underline{m}_{\underline{X}} \tag{10}$$

And the simple mean of $\underline{Y}^k$ it is turn in:

$$\hat{\underline{m}}_{\underline{Y}} = \frac{1}{M}\sum_{k=1}^{M}\underline{Y}^k. \tag{11}$$

By developing the sum we obtain an equality between the sample mean of $\underline{Y}$ and the theoretical mean of $\underline{X}$:

$$\hat{\underline{m}}_{\underline{Y}} = \frac{1}{M}\sum_{k=1}^{M}\underline{Y}^k = \underline{m}_{\underline{X}}. \tag{12}$$

Similarly the variance-covariance matrix of $\underline{Y}$ is:

$$\begin{aligned}
\Sigma_{\underline{Y}} &= \frac{1}{M}\sum_{k=1}^{M}\left(\underline{Y}^k - \hat{\underline{m}}_{\underline{Y}}\right)\left(\underline{Y}^k - \hat{\underline{m}}_{\underline{Y}}\right)^{\bullet} \\
&= \frac{1}{M}\sum_{k=1}^{M}\left(H\left(\underline{X}^k - \hat{\underline{m}}_{\underline{X}}\right) + \underline{m}_{\underline{X}} - \hat{\underline{m}}_{\underline{Y}}\right) \\
&\times\left(H\left(\underline{X}^k - \hat{\underline{m}}_{\underline{X}}\right) + \underline{m}_{\underline{X}} - \hat{\underline{m}}_{\underline{Y}}\right)^{\bullet} \\
&= \frac{1}{M}\sum_{k=1}^{M}H\left(\underline{X}^k - \hat{\underline{m}}_{\underline{X}}\right)\times H\left(\underline{X}^k - \hat{\underline{m}}_{\underline{X}}\right)^{\bullet} H^{\bullet} \\
&= H\Sigma_{\underline{X}}H^{\bullet} \\
&= \sqrt{\Sigma_{\underline{X}}}\left(\sqrt{\Sigma_{\underline{X}}}\right)^{-1}\left(\sqrt{\Sigma_{\underline{X}}}\sqrt{\Sigma_{\underline{X}}}\right)\left(\sqrt{\Sigma_{\underline{X}}}\right)^{-1}\sqrt{\Sigma_{\underline{X}}} \\
&= \Sigma_{\underline{X}}.
\end{aligned} \tag{13}$$

This transformation implies that the average of the sample $\underline{Y}^k$ is identical to the theoretical mean of $\underline{X}$ and the variance-covariance of $\underline{Y}^k$ is identical to the theorical variance-covariance matrix of $\underline{X}$. To execute this transformation is necessary to know the theoretical variance-covariance. Under these conditions, this transformation improves the precision of the estimates obtained from the Monte Carlo simulation.

Asian option with underlying average and stochastic interest rate

One way to consider the stochastic nature of interest rates is to model it through processes of mean reversion, such as models of short rate widely cited in the literature as are the models of Vasicek (1977) and Cox-Ingersoll -Ross (1985) among others. With respect to the calibration of these models are generalized method of moments and series of time with an alternative method to calibrate the parameters of interest rate models proposed by Overbeck and Rydn (1997).

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In practical terms this method consist in estimate a set of parameters within the conditional expectation of a stochastic process: $\mathrm{E}[X_t \mid X_{t-1}]$,then the estimated values are used as initial values to a maximum likelihood approach that accelerates the convergence to a global optimum.

On the other hand, one of the methodologies for determining the price of an option is through Monte Carlo simulation, the flexibility of the methodology proposed here is that with the assumption that the underlying price is driven by a Brownian geometric motion the payment function of the option arises a particular number of underlying paths that are generated and present values with the risk-free rate average trajectories.

This is how it can be determine the price of an European option, exotic options, among others, and is very useful when you have a closed formula. In the purposes of this paper it is assumed that the interest rate which is calculated with the hope pay function for an European call option and sell, and an Asian option price with underlying average, is stochastic and is model with a process of mean reversion.

The methodology is described for determining the price of an Asian option with underlying unpaid average of dividends through Monte Carlo simulation.

Suppose the price of the underlying S_t is conducted by a geometric Brownian motion with stochastic interest rate r_t, process driven by mean reversion and both processes are correlated as follows:

$$\frac{dS_t}{S_t} = r_t dt + \sigma dW_{S,t}, \tag{14}$$

$$dr_t = \kappa\left(\theta - r_t\right)dt + \sigma_r r_t^{\alpha} dW_{r,t}$$

Where $W_{r,t}$ is a Wiener process correlated with $W_{S,t}$, *e.g.*, $\mathrm{Cov}(dW_{r,t}, dW_{S,t}) = \rho dt.$ If $\alpha = 0$ Then the dynamics of the cut rate is driven by a Vasicek model considered $\alpha = 0.5$ for a process of a CIR type.

By making the analogy with stochastic volatility models, the parameters θ, κ and σ_r, are interpreted as a long-term rate, the rate of reversion to the long-term rate and the volatility of the variance of the interest rate (often referred to the volatility of volatility), respectively.

To make the simulation of both processes is necessary to generate trajectories with a structure given by:

$$dW = \begin{pmatrix} dW_{S,t} \\ dW_{r,t} \end{pmatrix} \sim N(0, \Sigma), \tag{15}$$

con:

$$\Sigma = \begin{pmatrix} \Delta t & \rho_{r,S}\Delta t \\ \rho_{r,S}\Delta t & \Delta t \end{pmatrix}. \tag{16}$$

To do this it is estimated L given that $\Sigma = LL'$ and simulated $dZ \sim N(0, I_2)$ to obtain $d\hat{W} = LdZ$, it is chosen a number for the partition of the term to maturity of the option, for example $N = 100$ and by the method of quadratic resampling generate dZ, involving $d\hat{W}$ for construction. Be μ_Z and Σ_Z, the

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theoretical mean and covariance matrix of dZ respectively:

$$\mu_Z = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{17}$$

y

$$\Sigma_Z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{18}$$

With the equations it is generated paths of the differential system of stochastic equations raised in (14).

The price of a call option with underlying average is given by the function of payment:

$$C(S_T) = \max\left(\frac{1}{T}\int_0^T S_\tau d\tau - K, 0\right) \tag{19}$$

And the price of a put option with underlying average is given by:

$$P(S_T) = \max\left(K - \frac{1}{T}\int_0^T S_\tau d\tau, 0\right). \tag{20}$$

If the underlying is driven by the system given in (14) then the algorithm to determine the prices of options of buying and selling is:

1) Generate $dW_{S,t}$ and $dW_{r,t}$ like:
   $$dW_{S,t}^{(k)} = Z_{S,t}^{(k)}\sqrt{\Delta t},$$
   $$dW_{r,t}^{(k)} = \rho Z_{r,t}^{(k)}\sqrt{\Delta t} + \sqrt{1-\rho^2}\, Z_{r,t}^{(k)}\sqrt{\Delta t},$$

2) Discretize the differential system of stochastic equations as:

$$r_{i+1}^{(k)} = r_i^{(k)} + \kappa\left(\theta - r_i^{(k)}\right)\Delta t + \sigma_r r_i^{(k)} dW_{r,t}^{(k)}$$

$$S_{i+1}^{(k)} = S_i^{(k)}\left(1 + r(t_i)\Delta t + \sigma_i^{(k)} dW_{S,t}^{(k)}\right)$$

$$i = 1, \ldots, N-1.$$

3) Define the arithmetic mean of the generated paths:
   $$\bar{S}^{(k)} = \frac{1}{N}\sum_{i=1}^{N} S_i^{(k)},$$
4) Calculate the option Price as:

$$C = \exp\left(-\int_0^T r_t dt\right)\frac{1}{M}\sum_{k=1}^{M}\max\left(\bar{S}^{(k)} - K, 0\right)$$

and

$$P = \exp\left(-\int_0^T r_t dt\right)\frac{1}{M}\sum_{k=1}^{M}\max\left(\bar{S}^{(k)} - K, 0\right)$$

Where M denotes the number of simulated paths and N the number of generated prices.

Application and analysis of results

In this section are calculated by Monte Carlo simulation Asian options prices with underlying average, with the underlying share price of AMX-L and compared with rates published in the MexDer newsletter the 25/10 / 2013. It is assumed that the rate of Interest of the option is driven by processes of mean reverting of Vasicek and CIR type, and to estimate the initial parameters we consider the maximum likelihood method proposed by Overbeck and Rydn (1997).

The sample of the interest rate TIIE28 to calibrate the models includes the October 25, 2012 to October 25, 2013 With data obtained from the website of the central bank estimates. The Vasicek and CIR model parameters are as follows:

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Summary table of parameter estimation of Vasicek and CIR models by the maximum likelihood method

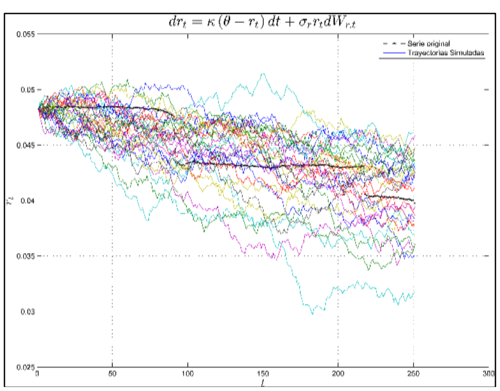
Parameters	**Vasicek**	**CIR**
κ	0.749896	0.809136
θ	0.033233	0.034069
σ	0.004879	0.023153
No. Obs. n	251	251
Likelihood Ratio	6.643470	6.641966

Table 2

The results of the above table can verify that the Feller condition is fulfilled: $2\kappa\theta > \sigma^2$ in both cases is important since it implies that the process lead a positive variance.

Graph 3 shows the original series of the risk-free rate and thirty simulated trajectories with the parameters given by the Vasicek model and Graph 4 with parameters given by the CIR model. In both cases the trend toward lower rate is observed, the original series also shows two significant changes: the first of 4.7550% to 4.3450% on 11/03/2013 and the second from 4.3075% to 4.0570%.

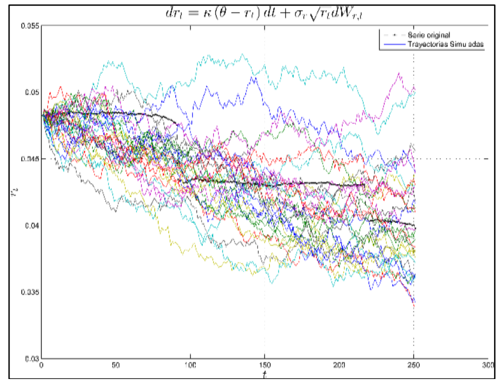
Original series and thirty simulated trajectories parameters calibrated with the Vasicek model



Graphic 3

Souce: Own elaboration

Original series and thirty simulated trajectories parameters calibrated with the CIR model



Graphic 4

Source: Own elaboration

The results of the application are shown in the Appendix. Graph 5 shows results for terms ranging from T = 56, 147, 238 and 329 days, option prices obtained with the Vasicek model, and Graph 6 for option prices obtained with the CIR model.

The number of trajectories that was simulated to determine the price of the options was a hundred-thousand. An important result is that by comparing prices of European options, with both simulated and published by MexDer with their Asian counterparts, Asian options prices are lower in the case of buying and selling options inside the money. For deeper options in the money the difference is smaller.

It is observed that as the term to maturity of the option is greater, the difference increases. For simulated purchase option prices the differences are greater compared to rates published by MexDer. To put options is why the simulated prices were lower in all cases.

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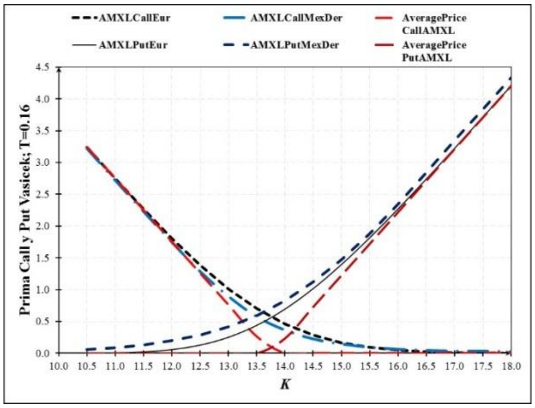
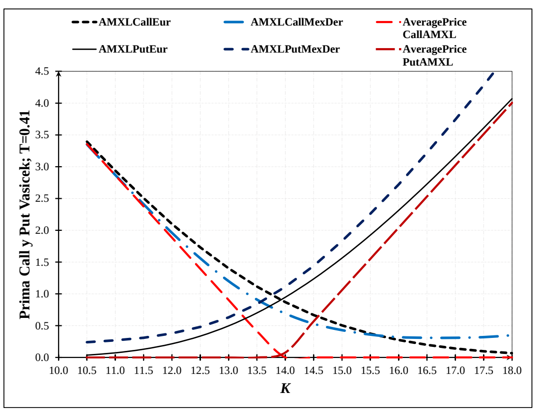
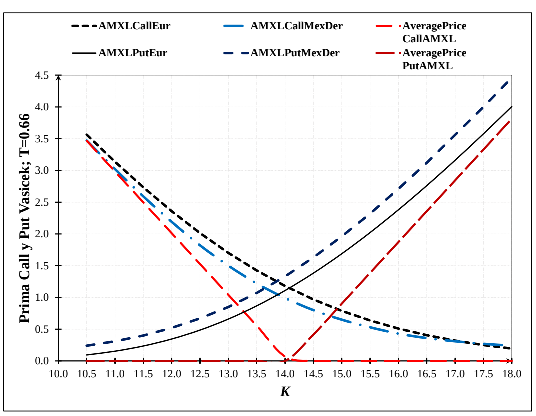
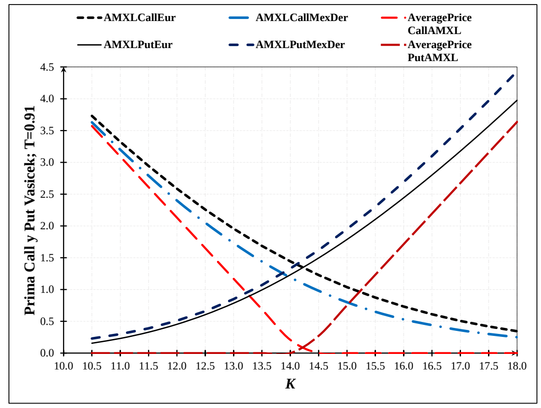
Comparing prices of European options and underlying average, with parameters calibrated with the Vasicek model





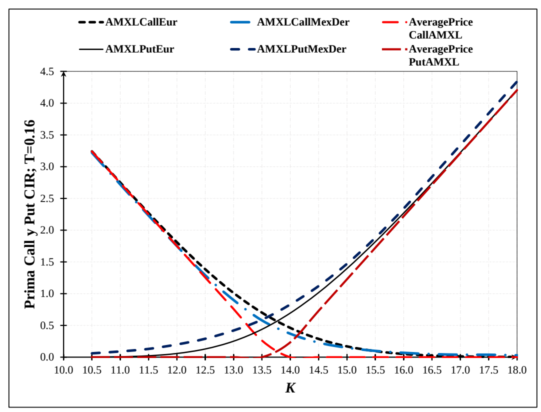


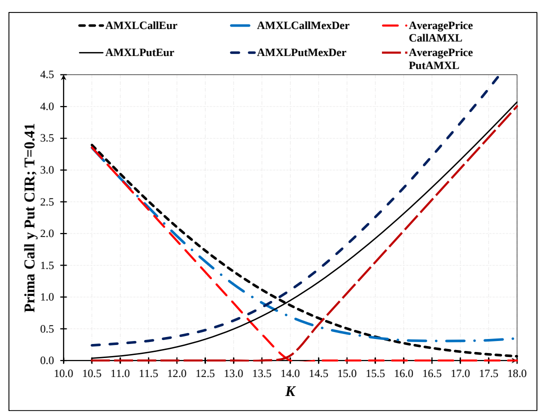


Graphic 5

Source: Own elaboration

Comparing prices of European options and underlying average, with parameters calibrated CIR model





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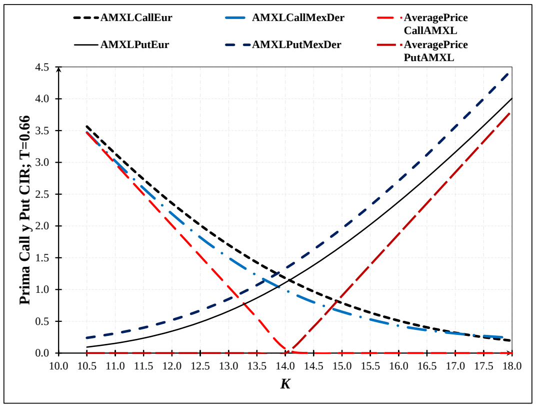

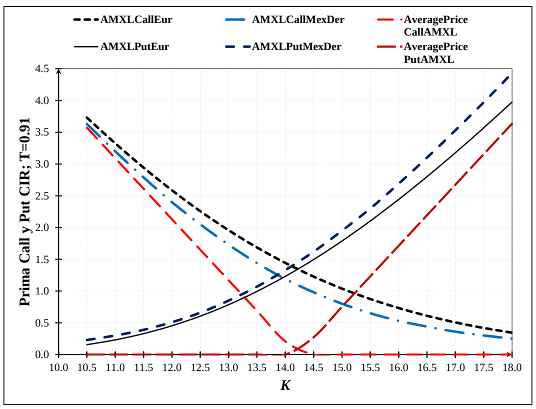

Graphic 6

Source: Own elaboration

Conclusions

Asian options are options where the underlying is the average price for a period of time. With this feature they have a lower volatility and therefore are cheaper compared to European options.Are mainly traded on currencies and commodities that have low trading volumes.

They were originally used in 1987 when the office of Trust Bank in Tokyo used them to determine the price of options on the average price of a barrel of oil, and therefore the option is known as "Asian”.

Asian options can be classified into three categories: arithmetic and geometric mean and both can be weighted in various ways, in which a certain weight is applied to each underlying where the average is calculated. This can be useful to determine the average of a sample with a skewed distribution. An additional feature of Asian options is that the underlying may be the average price or the strike price of the underlying average take over the contract period.

In this research through Monte Carlo simulation and the method of quadratic resampling of Barraquand (1995) were determined prices of European options for buying and selling, and option prices of purchase and sale with average core, also known as average price (call and put) also assumes that the interest rate is stochastic and driven by a process of mean reversion type Vasicek and CIR.

The results show that by comparing prices of European options, with both simulated and published by MexDer with their Asian counterparts, Asian options prices are lower in the case of buying and selling options inside the money. For deeper options inside the money the difference is less, in both cases it was observed that as the term is mature the option is greater. To put options simulated prices were lower in all cases.

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