Article **Journal of Quantitative and Statistical Analysis**
June 2022, Vol.9 No.24 15-22

Thermodynamic analysis of a Stirling cycle with nuclear heat source for aerospace applications

Análisis termodinámico de un ciclo Stirling con fuente de calor nuclear para aplicaciones aeroespaciales

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DOI: 10.35429/JQSA.202.24.9.15.22 Received January 20, 2022; Accepted June 30, 2022

Abstract

The main requirement for the development of aerospace missions is the energy supply, for example, 10 kWe would be required for the day and 9 kWe for the night during the first phase of construction of a human settlement on the moon. To satisfy the energy demand, it was proposed the use of heat due to a nuclear fission reaction, coupled to a Stirling engine as a dynamic power converter. The Stirling engine is used since it has less mass, a smaller heat sink area and a longer useful life compared to another type of power converter, thus being the most suitable for coupling with the Optimized Evolutionary Growth Lunar nuclear reactor, which will be the thermal power source of the electrical power generation system presented in this article. In this article, it is shown the thermodynamic analysis that involves the transport of heat from the nuclear reactor to helium as the working fluid of the dynamic power converter. The initial parameters are obtained for the analysis: temperature, pressures and volumes, which will allow us to carry out the mathematical modeling of the Stirling Method (Ideal). As results, a comparison is presented between the variations of proposed parameters with the purpose of determining the behavior of the useful work and the electrical power in the system, evaluating the compression ratio, the angular velocity and the initial pressure.

Stirling engine, Nuclear reactor, Thermodynamic analysis

Resumen

El principal requerimiento para el desarrollo de misiones aeroespaciales es el suministro energético, por ejemplo, se requeriría 10 kWe para el día y 9 kWe para la noche durante la primera fase de construcción de un asentamiento humano en la luna. Para satisfacer la demanda energética se propone el aprovechamiento del calor debido a una reacción nuclear de fisión, acoplada a un motor Stirling como conversor de potencia dinámica. Se utiliza al motor Stirling ya que tiene menor masa, menor área de disipador de calor y mayor vida útil en comparativa con otro tipo de conversor de potencia, siendo así el más apto para el acoplamiento con el reactor nuclear Lunar de Crecimiento Evolutivo Optimizado, el cual será la fuente de potencia térmica del sistema de generación de potencia eléctrica presentado en este artículo. En este trabajo se presenta el análisis termodinámico que involucra el transporte de calor del reactor nuclear al helio como fluido de trabajo del convertidor de potencia dinámica. Para el análisis se obtienen los parámetros iniciales: temperatura, presiones y volúmenes, los cuales nos permitirán realizar el modelado matemático el Método Stirling (Ideal). Como resultados se presenta una comparativa entre la variación de parámetros propuestos, con la finalidad de determinar el comportamiento del trabajo útil y la potencia eléctrica en el sistema, evaluando la relación de compresión, la velocidad angular y la presión inicial.

Motor Stirling, Reactor nuclear, Análisis termodinámico

Citation: DÍAZ-ESPINOZA, Gerardo, GALLARDO-VILLARREAL, José Manuel and VALLE-HERNÁNDEZ, Julio. Thermodynamic analysis of a Stirling cycle with nuclear heat source for aerospace applications. Journal of Quantitative and Statistical Analysis. 2022. 9-24:15-22.

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Introduction

NASA is developing different missions that involve three space applications: Rovers and Satellites, Propulsion, and Settlement. This last application suffers from the problem of how electrical power is supplied, an example of this is the energy consumption demanded for the first stage of lunar habitation established in Power System Requirements and Definition for Lunar and Mars Outposts (I), which is 5-6 kilowatts electric (kWe) (I).

In recent years, the use of solar energy has been implemented as an electrical power supply, however, the use of these mechanisms has disadvantages due to the long duration of the night (14 Earth days) involving the solar cells do not work during that period of time, likewise, during the day, the system is subjected to a large amount of heat causing the efficiency decreases.

It begins to analyse the advantages of using a nuclear source as a thermal power supply, one of them is to provide constant energy, which is a necessity for human life support systems, rover recharging and resource extraction, unlike solar energy which requires the use of energy storage devices increasing unwanted mass to the system (I).

Therefore, the selection of a nuclear reactor with characteristics concerning the feasibility of use on the moon has been made, in this case, the Lunar Evolutionary Growth Optimised (LEGO)(II) nuclear reactor represents a system capable of providing an initial supply of useful thermal power for power conversion, reaching the electrical power required to satisfy the energy need.

The thermal power supplied by the LEGO nuclear reactor is 24 kWt (II), using Uranium Dioxide as fuel and implementing metallic liquid sodium as coolant, transporting the heat to a first exchanger where potassium is located, generating a phase change from liquid to vapour that will transport the heat to the power converter.

It has been decided to implement a dynamic power converter for heat utilisation, involving high efficiency, lower mass for transportation and sufficient power supply required, being the Stirling engine the most viable system compared to other types of converters.

Therefore, the contribution of this article is to perform the thermodynamic analysis of the Stirling engine to have an approach to the behaviour of the cycle in ideal and ideal geometric conditions, being the basis of a project in which real conditions of heat transfer and behaviour of working fluids in the coupling of the nucleoelectric system will be determined.

Then, the sections of this work are presented. For the first section, the initial parameters of the Stirling engine system (XI) are determined.

Finally, the modelling of the Stirling engine is established, based on the First Order Model corresponding to the Ideal Stirling Cycle where the initial parameters will be used to carry out the analysis and verification of all the characteristics involved in obtaining the electrical power.

Methodology

The following are the procedures to be used to carry out the research presented (Fig. 1).

I. Define the initial temperature of the working fluid in the expansion zone of the engine due to the thermal power released by the nuclear reactor.

II. Obtain initial parameters that will be used in the modelling of the Stirling engine.

III. To analyse the thermodynamic cycle for a Stirling engine based on the Carnot Cycle, generating the first order model Ideal Stirling Model.

IV. To evaluate the electrical power and efficiency of the cycle by comparing the values proposed in the model.

ISSN: 2410-3438

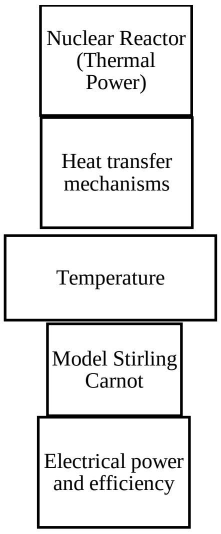


Figure 1 Development of the methodology

Modelling Considerations

Thermal Power and Heat Transfer

The Optimised Evolutionary Growth Lunar nuclear reactor is proposed for this analysis as the heat source for this system, its thermal power behaviour was the reference for this article, using 24 kWt as the constant value in the system to obtain the temperature of the helium as the working fluid in the Stirling Cycle (II).

Ideal conditions were used to determine the initial parameters in the system, with conduction and convection heat transfer mechanisms being the means to determine these values.

$$Q = \frac{2\pi Lk(T_{PI}-T_{PE})}{\ln\left(\frac{r2}{r1}\right)} \tag{1}$$

$$Q = 2\pi r_1 Lh(T_{PE} - T_\alpha) \tag{2}$$

$$Q = mC_p\left(\frac{dT_\alpha}{dt}\right) \tag{3}$$

Where k is the value of the conductivity of the material in the system, A=2πrL the respective area in each section, Tpi the initial reactor temperature, Tpe the external wall temperature, Tα the fluid temperature, m the total mass of the system, h the convective heat transfer coefficient and Cp the specific heat (VII).

The idealisation of the system is schematized below.

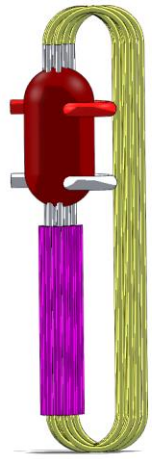


Figure 2 Idealisation of the coupled system

Fig 2 shows the section of the lunar nuclear reactor denoted by the pink colour, the high conductivity piping in white and yellow, the first heat exchanger where the potassium is stored in red.

Sodium as the main working fluid transports the heat through the nuclear reactor, then the thermal power is delivered to the first heat exchanger where potassium is stored.

Finally, the helium in the Stirling engine section receives the heat transported by the potassium due to the first heat exchanger, being this temperature the one used in the cycle to be developed (II).

The selection of 4 proposed parameters is included in which, as first parameter, is the initial pressure that will establish the first process due to the obtaining of the specific volume of the thermodynamic cycle respectively.

As a second parameter, the compression ratio is selected, which based on the Stirling Engine Design Manual (X) shows that there can be a variation of this ratio in an interval of 1.8 to 3 for a Stirling engine, being the optimum values to determine the change in the volume in the compression zone.

Subsequently, the angular velocity that will be included in obtaining the resulting mechanical power in the system will be selected and a comparison will be made to determine the optimum values (IX).

Obtaining the initial cycle parameters

The proposed parameters in table 1 are:

Parameters proposed	
Initial Pressure (kPa)	200
Compression Ratio	1.8
Angular velocity (rpm)	1500
Geometric Volume (m^3)	0.009

Table 1 Proposed parameters for Stirling engine modelling

The ideal gas characteristics are also established to determine the initial working parameters of the Stirling cycle, including equation (1), (2) and (3) to determine the value of the working fluid temperature due to the thermal power of the reactor (Table 2). (Table 2)

$\dot{Q}$ (kWt)	T_1 (K)	P_1 (kPa)	$V_1 \left(\frac{m^3}{kg}\right)$	T_3 (K)
24	973	200	10.07	600

Table 2 Initial parameters for Stirling engine modelling

As shown in equation (4), based on the geometric volume parameter, the total mass in the system was obtained by referring to the specific volume of expansion, the largest area.

$$m = \frac{V_g}{V_e} = 0.00049\ kg \tag{4}$$

Modelling of the Stirling engine

For the modelling of the Stirling engine, the geometrical configuration type Beta was considered, where the cycle is driven by a piston and a displacer operating concentrically in the same cylinder (X).

Taking into account these characteristics, this type of engine was considered for its aerospace application, since, through an arrangement to the mechanical system, which interconnects the expansion and compression section, it could be hermetically sealed avoiding the leakage of the working fluid, being this a desirable feature for an implementation in the lunar environment (V).

The working fluid in this system is helium due to its low molecular weight and high thermal conductivity compared to other working fluids used in this system. other working fluids used in this type of configuration (IV).

As a first approach to the mathematical and thermodynamic model of the Stirling engine, the ideal modelling is established based on the Carnot cycle made up of the four processes shown in table 3.

Process	
1-2	Isothermal Expansion
2-3	Isothermal Expansion
3-4	Isothermal Compression
4-1	Isentropic Compression

Table 3 Thermodynamic processes for Carnot method

In contrast to the Carnot modelling, the Ideal Stirling model replaces the isentropic processes with two regeneration processes, in which heat is transferred to a thermal energy storage system and subsequently delivered to another process in the system.

Ideal Stirling model

The modelling of the Ideal Stirling Cycle is described in Table 4, and allows us to determine the thermodynamic behaviour of the Stirling engine, based on the law of conservation of mass, considering ideal conditions without losses and reversible.

Process	
1-2	Isothermal Expansion
2-3	Regenerative Cooling
3-4	Isothermal Compression
4-1	Regenerative Heating

Table 4 Thermodynamic processes for ideal Stirling method

Three main equations have been established to carry out the modelling of the Ideal Stirling cycle, the first equation is the one corresponding to the law of conservation of energy shown in equation (5) which establishes the change in internal energy based on heat and work in the whole system (VII).

$$\Delta U = Q - W \tag{5}$$

The second equation of the model corresponds to the work in a process of isothermal compression or expansion, as shown in the following equation:

$$W = \int pdV \tag{6}$$

ISSN: 2410-3438

Finally, the third equation used corresponds to the ideal gas equation (7), which is useful to determine the changes in specific volume, pressure and temperature, respectively.

$$PV = mRT \tag{7}$$

With these three equations, the values of each process of the Thermodynamic Cycle are determined.

Stirling Thermodynamic Cycle

a) Isothermal Expansion

For the isothermal expansion process, there is no variation in the internal energy because the temperature remains constant as shown in:

$$\Delta U_{1-2} = mC_v(T_2 - T_1) = 0, T_2 = T_1 \tag{8}$$

C_v being the specific heat at constant specific volume, m the total mass in the system T_2 the temperature in the compression zone and T_1 the temperature in the expansion zone.

It is also implied that the gas does work and therefore has to absorb an equal amount of energy from the heat supply to keep its temperature constant.

$$Q_{1-2} = W_{1-2} = mRT_e \ln\left(\frac{V_1}{V_2}\right) \tag{9}$$

where m is the mass in the system, R is the ideal gas constant, V_1 is the specific volume in the compression zone and V_2 is the specific volume in the expansion zone.

b) Regenerative Cooling

In the regenerative cooling section, it is stated that there is no work in this process because the specific volume in the process remains constant.

$$W_{2-3} = mRT_c \ln\left(\frac{V_2}{V_3}\right) = 0 , V_2 = V_3 \tag{10}$$

is shown as a process in which the gas gives up heat, decreasing its internal energy and therefore its temperature decreases.

$$Q_{2-3} = \Delta U_{2-3} = mC_v(T_3 - T_2) \tag{11}$$

c) Isothermal Compression

Being the process where the gas is compressed at the constant temperature of the compression zone, from the specific volume V_2 to the final specific volume V_1.

$$\Delta U_{3-4} = mC_v(T_4 - T_3) = 0, T_4 = T_3 \qquad (12)$$

The work for this process is due to the low-pressure gas, being less than during the expansion process.

$$Q_{3-4} = W_{3-4} = mRT_e \ln\left(\frac{V_3}{V_4}\right) \qquad (12)$$

It must supply the same amount of heat to keep the temperature constant compared to the expansion section.

d) Regenerative Heating

The behaviour in this process is at constant volume, causing zero work in this section because of

$$W_{4-1} = mRT_c \ln\left(\frac{V_1}{V_4}\right) = 0 \quad , V_1 = V_4 \tag{13}$$

For the heat of temperature change at constant specific volume, the increase in internal energy is established, therefore:

$$Q_{4-1} = \Delta U_{4-1} = mC_v(T_1 - T_4) \tag{14}$$

Making use of the values obtained in each process, we proceed to determine the efficiency and mechanical power based on the following equations:

$$\eta = \frac{W_T}{Q_e} \tag{15}$$

W_T being the network in the variation of the expansion and compression work, and Q_e the heat in the compression zone.

$$Pot = \frac{W_T * \dot{n}}{n_{rev}} \tag{16}$$

Where $\dot{n}$ is the angular velocity established at the beginning of this article and n_rev is the number of revolutions per unit cycle, for the case of the Stirling engine the value is 1 rev/cycle (VII).

ISSN: 2410-3438

Results and discussion

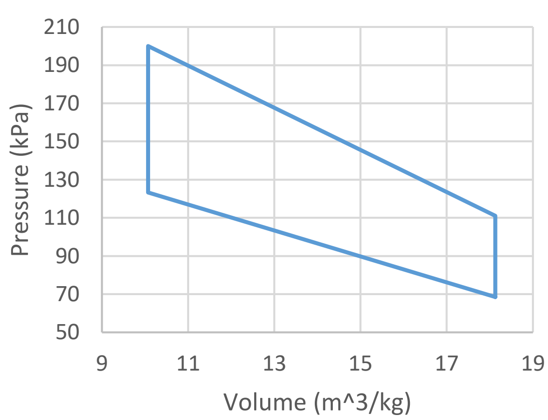
In the pressure-volume diagram in Graph 1, the four processes corresponding to the thermodynamic cycle can be observed.

For section 1-2, the decrease of the initial pressure (Table 1) is noted, reaching a point where the specific volume will have no variation.

When it reaches 111 kPa, the following process takes place, the specific volume remains constant while the temperature decreases, this is due to regenerative cooling, where the regenerator absorbs heat and stores it.

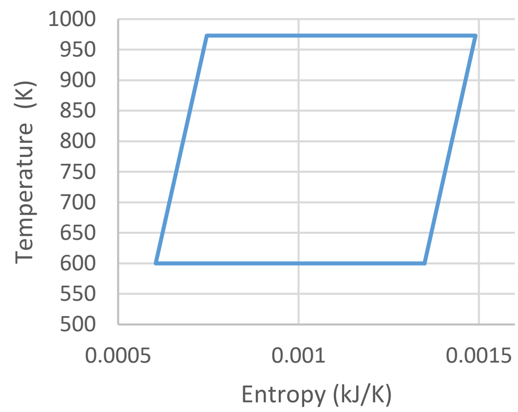
When the minimum pressure is reached, the compression process takes place due to the regression of the piston in the expansion zone, so the specific volume decreases and the pressure increases.

Finally, when the maximum specific volume point is reached, heating occurs due to the influence of the regenerator, which delivers the heat absorbed in the process 2-3, generating an increase in temperature, which leads to the restart of the cycle.



Graphic 1 Ideal PV Diagram

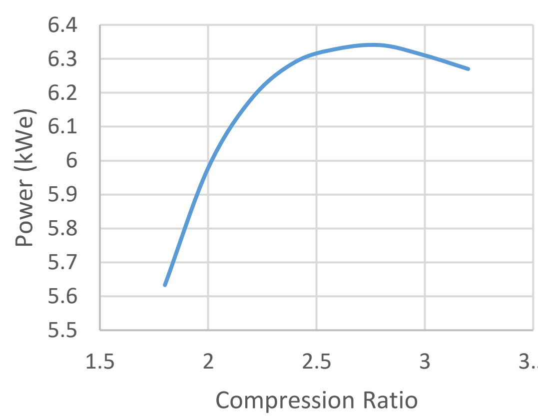
For Graphic 2, the bounded area represents the resulting heat in the system with respect to the entropy change, observing in the cooling process a decrease in temperature and in the regenerative heating of the fluid an increase.



Graphic 2 Ideal TS Diagram

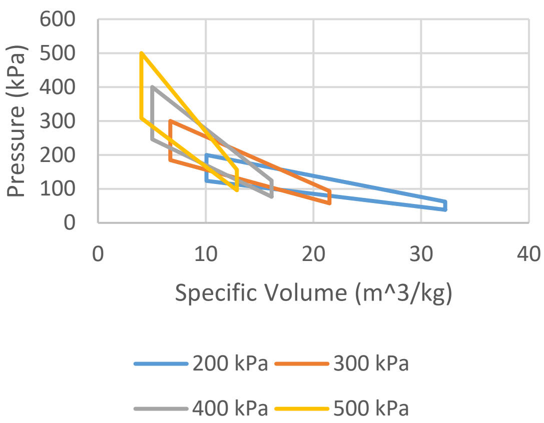
In Graphic 3, it can be seen that the power required by the Lunar Evolutionary Growth Reactor has been reached, being 5.63 kWe, which allows us to verify that the mathematical modelling is correct (II).

By the variation in the compression ratio that, as a respective increase, the power also increases, however, increasing the compression ratio causes the decrease in power, therefore, it was proposed to fix the value of compression ratio and make the variation in other parameters.



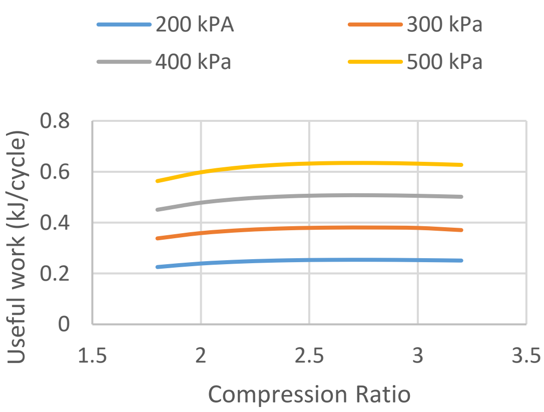
Graphic 3 Compression Ratio Variation

For Graphic 4, the variation in the value of the pressure in the expansion section was carried out, observing the considerable increase in the useful work, being a 33% increase by increasing 1 atm, however, it has been decided that the use of the working fluid is at low pressure due to present structural advantages in terms of chemical compatibility with the material and decreasing the possible leaks in the system being more manageable for the user.



Graphic 4 Expansion zone pressure variation

According to Graphic 3, it is established that if there is an increase in the compression ratio there will be an increase in the power delivered by the system, however, it is determined in Graphic 5 that, if the compression ratio continues to increase, the performance of the system decreases, so it is necessary to establish an optimum point by limiting the increase in the expansion volume to 2.8.



Graphic 5 Variation of useful work due to compression ratio

The variation of the useful work at different pressures was reviewed, as it can be observed, the increase in the initial pressure causes an increase in the value of the useful work, however, the variation of the compression ratio was maintained, causing the same behaviour of performance decrease.

Conclusions

The verification of this model was based on the results established in the article A Basic LEGO Reactor Design for the Provision of Lunar Surface Power. United States (II), as it provides an electrical power supplied by the nuclear reactor of 5.63kWe.

Based on the characteristics of high efficiency, structural advantages, size, weight, lifetime and use of a regenerative system, the Stirling Cycle engine was determined to be the most viable power converter for the problem set out in this paper.

The optimal results of system behaviour with the characteristics of the initial parameters proposed were arrived at through mathematical modelling.

Taking into account that the electrical power requirements are not so high, it was decided to maintain a relatively low angular velocity for this system based on the analysis of the results in section 5 of this article.

However, it was observed that increasing the compression ratio at a certain point the power starts to decrease presenting low efficiency, which allows us to analyse the optimal compression ratio value for the system.

The implementation of a Thermodynamic Stirling Cycle showed the behaviour that the proposed variables can be modified according to the user's needs.

Finally, reference is made to the previous results being the optimal system parameters for the need of this article; for future projects they will be taken as a basis to develop a real geometrical analysis and the thermo-hydraulic modelling of certain sections of the above mentioned nuclear power system (IV).

Funding

The present work has been funded by CONACYT Grant Number 926199.

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