This paper examines seasonality effects on both the return and volatility of a GARCH family model for the French CAC-40 daily index returns. Four calendar effects — day-of-the-week, turn-of-the-month, month-of-the-year and holiday effect — are simultaneously examined. We examine the changes in inferences that might occur when the error terms of descriptive models modeling volatility are specified under different error distributions: normal, Student’s-t, generalized error distribution and double exponential distribution. The usefulness of the in-sample significant estimated seasonality patterns for out-of-sample forecasts in return and volatility is also examined. We find that the few significant seasonality patterns in descriptive models, in the mean and conditional volatility equations, are sensitive to the underlying distributions of the error term. Additionally, the significant estimated effects are not useful in explanatory models and do not introduce predictive ability against the random walk model.

**Seasonality effects, conditional volatility, error distributions, stock market.**

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Introduction

The existence of seasonality effects has been documented over the last three decades in the financial markets. These studies challenged the assumptions of the dominant theory (Efficient Market hypothesis) and suggested alternative explanations for possible regularities in prices both due to the behavior of investors and institutional arrangements. However, various empirical studies have reported a decline on seasonality over time. Rationally, as these anomalies are relatively easy to exploit, if they were regular, predictable and had any economic significance after accounting for transactions costs it would be expected that they have weakened over time. Although not contradicting an enlarged efficient market view, the eventual seasonality effects, albeit statistically but not economically significant, could be due, for instance, to the higher transactions costs over the potential gain, the \textit{ex ante} uncertainty on whether seasonality effects will materialize and external arrangements to the market. Yet, also the robustness of these significant effects remains a controversial issue either in the stock, foreign exchange and forward markets. Connolly (1989, 1991), after adjusting \textit{t}-values for sample size with a Bayesian approach shows that evidence of day-of-the-week effects reported in earlier studies disappears. Chang et al. (1993) report that Connolly’s evidence holds for the U.S. and other international markets. Hsieh (1988) notes that evidence of the day-of-the-week effect may be illusory if not properly accounting for the non-normality and volatility clustering observed for spot foreign exchange rate distributions. As such this paper is concerned with the “weak form efficiency”, i.e., whether asset prices reflect the past history of prices including seasonality effects.

The focus of the study is on the seasonal patterns in conditional mean and volatility equations subject to various error distributional assumptions.

The robustness of seasonality effects is often called into question because many previous studies have generally ignored the econometric issues and based their analysis mainly on the results of Ordinary Least Squares method (OLS) which do not account for the stylized facts of financial time series (i.e. non-normality and volatility clustering). The distribution of stock returns and hence the error term of regression models is also a key issue in examining the seasonality. If the true error distribution is considerably fatter-tailed than the normal, the distribution assumed in much of the previous papers, the null hypotheses of no seasonality effect is more likely to be rejected than the chosen significance level would indicate. Studies failing to take into account stylized facts of financial time series may report effects that do not exist. Fama (1965) suggest that the variance of returns might be infinite and best modeled by a stable paretian distribution. Blattberg and Gonedes (1974) and Jansen and de Vries (1991) argue that daily stock returns could be adequately modeled by a fat-tailed distribution such as the Student $t$-distribution. The time-varying volatility and volatility clustering are also stylized facts in daily stock returns. Much of the literature focuses on non-linear models of the GARCH (General Auto-Regressive Conditional Heteroscedasticity) family to explain the volatility (variance) of prices. Baker et al. (2008) report that using GARCH models to test for the day-of-the-week effect on both the mean and volatility are not suitable when it is assumed that the returns follow a normal distribution.
Though evidence exists for the main seasonality effects for the mean returns, only limited evidence exist for similar effects on conditional volatility. Berument and Kiymaz (2001) test for the day-of-the-week effect on conditional volatility for the S&P 500 index assuming a GARCH specification with a normal distribution for stock returns. They show that volatility varies by the day of the week with the highest volatility observed on Fridays. Likewise, using a similar framework with a normal distribution, Kiymaz and Berument (2003) test for the day-of-the-week effect on mean, volatility and transaction volume for the major global stock markets indexes and find that the effect is present in both return and volatility equations. Choudhry (2000) provides evidence of the day-of-the-week effect in emerging Asian countries using a GARCH model that assumes the error distribution follows a conditional Student-t density function. Baker et al. (2008) using a GARCH specification report that the day-of-the-week effect in both mean and volatility for the S&P/TSX composite price index from the Toronto Stock Exchange is sensitive to the error distributional assumptions.

The purpose of this paper is to simultaneously examine a range of seasonality effects in both the mean and conditional variance in the CAC-40 stock return index subject to various error distributional assumptions. The study makes a distinction between descriptive and explanatory study. The first part involves a descriptive study in which the interest is in determining the evidence about seasonal patterns in the mean and conditional variance. The second part involves an explanatory study where the intention is to predict future mean and return volatility.

The seasonality patterns could manifest itself in the estimation period but they should be considered important only if its inclusion in the model result in better forecasts. To account for autocorrelation, non-normality and volatility clustering, we use an AR(K)-EGARCH(p,q) model under normal as well as three additional error distributions that are fatter-tailed than the normal to accommodate the stylized facts in financial time series: Student-t, generalized error distribution (GED) and double exponential error distribution (DED). This allows us to test for the robustness of seasonality effects to error distributional assumptions in both the conditional mean and volatility. Our general null hypothesis is that the various seasonality effects are robust to error distributional assumptions in both returns and conditional volatility.

This study contributes to the literature in two ways. First, we consider at the same time various seasonality effects in both the mean and conditional volatility using several error distributional assumptions. Second, we consider the descriptive study separated from the predictive study using a non-overlapping sample for each analysis. If there are seasonality patterns in the mean and/or variance they should be considered important only if its inclusion in the explanatory model result in better forecasts. The framework analysis used in this study is similar to that used by Baker et al. (2008). However, these authors only provide a descriptive study, analyze only one seasonality effect and do not perform an explanatory study on the usefulness of the significant in-sample seasonal patterns for forecasting returns and volatility. The findings show that the few significant seasonality patterns in descriptive models, in the mean and conditional volatility equations, are sensitive to the underlying distributions of the error term.
Also, the significant estimated seasonality effects are not useful for forecasting purposes and do not add predictive ability against the random walk model.

The paper is structured as follows. Section 2 reviews the literature of seasonality effects in mean return and conditional volatility under alternative error distributional assumptions. In Section 3 we provide a description of the data series, we analyse their distributional features and statistical tests for the homogeneity of the means and variances are conducted. In Section 4 we start by estimating an OLS model with the various seasonality effects to examine the evidence of significant patterns. Then an AR(K)-EGARCH(1,1) model with the various seasonality effects in the mean equation is estimated under alternative error distributions and an analysis of sensitivity of inferences is carried out. In Section 5 an AR(K)-EGARCH (1,1) model with seasonality effects included in the mean and conditional volatility equations are estimated and an analysis of sensitivity of inferences to alternative error distributions is conducted on the two equations. In section 6 we analyse the usefulness of the significant estimated seasonality effects, in the return and conditional volatility equations of the above descriptive models, to forecast out-of-sample the return and volatility. Finally, section 6 presents a summary and conclusions.

**Literature Review**

The most common seasonality effects in financial markets are the January effect (also termed turn-of-the-year or month-of-the-year effect).

The weekend and day-of-the-week effects; the turn-of-the-month effect (or the monthly effect) and the holiday effect. The literature contains many studies on the above cited effects on the mean returns. A number of hypotheses have been put forward to explain the presence of such seasonality.

The *January effect* refers to the higher returns in January reported by many researchers in various markets (Gultekin and Gultekin 1983; Arsad and Coutts 1997; Mehdian and Perry 2002; Al-Saad and Moosa 2005). Initially Rozeff and Kinney (1976) and Keim (1983, 1986) found this effect to be particularly large for returns on small stocks using returns from US stock portfolios. For the US market, the most popular explanation for higher January returns is the tax-loss selling hypothesis associated with the payment of tax bills each December (end of the (US) financial year): investors sell stocks with losses in December to qualify for a tax-loss and then invest the available funds in January. Several papers found empirical support for the tax-loss selling hypothesis (Dyl and Maberly 1992; Griffiths and White 1993; and Agrawal and Tandon 1994).

There is a difference between the weekend and the day-of-the-week effects. In the former, stocks exhibit lower returns between Friday and Monday closing (Agrawal and Ikenberry, 1994; Wang et al., 1997). In the second, returns on some trading days of the week are higher than others (Chang et al., 1993; Kamara, 1997; Chang et al., 1998).

Alternative explanations for the January effect exist. Odgen (1990) argues that the effect stems from seasonal cash received by investors. Miller (1990) suggest that year-end time pressures cause investors to postpone purchases until January, while sales in December are more likely for tax-loss reasons and because deciding to sell stock already owned takes less time than deciding what new stocks to buy. Lakonishok *et al.* (1991) report evidence consistent with the hypothesis that institutions often sell their losers in December to window-dress their end-of-year reports.
The January effect has also been found in other countries. In a study of the stock markets in 17 major industrialized countries over the period 1959-79, Gultekin and Gultekin (1983) found that significant differences in the month-to-month mean returns were present in 12 countries.

The day-of-the-week effect, initially studied in US markets (French 1980; Gibbons and Hess 1981), refers to the finding that Monday returns are, on average, negative and lower than for the rest of the week. A number of studies have focused on and reported evidence on the day-of-the-week effect (see, for example, among others, Jaffee and Westerfield 1985; Thaler 1987; Agrawal and Ikenberry 1994; Arsad and Coutts 1997; Keef and Roush 2005). An explanatory hypothesis is that more stocks go ex-dividend on Mondays, thereby lowering prices and returns. Some have suggested that stock returns could be lower on Mondays if firms typically wait until weekends to release bad news. Other work casts some doubt on the robustness of the weekend effect. Connolly (1989) argues that previous findings depend heavily on the assumption that returns are normally distributed with a constant variance. Using estimators that are robust with respect to violations of these assumptions, he finds much weaker evidence of a weekend effect, particularly after 1975. Chang et al. (1993), using procedures similar to Connolly, also report little evidence of an effect for a portfolio of larger companies’ stocks for the period 1986 to 1990. Some recent studies have also shown a decline in the Monday effect in the US markets (Chen and Singal 2003; Marquering et al. 2006).

The turn-of-the-month effect (TOM), first reported by Ariel (1987) in US markets, is the concentration of positive stock returns in the last trading day and the first nine trading days of each month. Various explanations have been put forward: a portfolio rebalancing, a month-end cash flow and company announcement hypotheses. Ariel could not account for this effect by the turn-of-the-year effect, dividend patterns, or higher return volatility at the beginning of months. He suggests systematic purchasing by pension funds at the turns of months as a possible explanation. Ogden (1990) attributes the effect to the temporal pattern of cash received by investors, while Jacobs and Levy (1988) attribute it to investors’ desires to postpone decisions until the beginnings of periods. Kunkel et al. (2003) carried out an extensive study of this effect in major global stock markets. They examine the evidence of the TOM pattern in 19 country stock market indices and found that the 4-day turn-of-the-month period accounts for 87% of the monthly return, on average, across countries, in the equity markets of 15 countries where this pattern exists.

The holiday effects allow the mean returns to be different on the day before a holiday and the day after. The pre-holiday effect is also associated with Ariel (1990). He reports that returns on days before such standard holidays as Christmas or Labour Day have been about 10 times the return on other days. Pettengill (1989) also reports evidence of high returns on pre-holidays. Lakoishok and Smidt (1988) report similar evidence over a much longer time period. Kim and Park (1994) also find higher pre-holiday mean returns for U.K. and Japanese as well as U.S. stocks, and that the effect in the first two was independent of the US markets.

Seasonality in profit announcements and tax deadlines hypotheses are also suggested.
Vergin and McGinnis (1999) examined the pre-holiday effect and found that this effect has disappeared for large firms but persists for small firms, though on a scale unlikely to exceed transaction costs. Thus, the gathered evidence for holiday effects suggests that higher than normal returns occur before a holiday, because of increased activity, and lower returns after the holiday. However, in recent years, the evidence for these effects has diminished.

Although the focus of the above studies has been the seasonal pattern in average returns, many empirical studies have investigated the behavior of the stock price series in terms of volatility using variations of GARCH models (French et al. 1987; Baillie and DeGennaro 1990; Nelson 1991; Glosten et al. 1993). French et al. (1987) examine the relationship between prices and volatility and report that the unexpected returns are negatively related to unexpected changes in volatility. Nelson (1991) and Glosten et al. (1993) report that positive (negative) unanticipated returns result in reduction (increase) in conditional volatility. Baillie and DeGennaro (1990) do not report evidence of a relationship between the average returns on equity portfolios and the variance of these returns. Corhay and Rad (1994) and Theodossiou and Lee (1995) investigated the behaviour of stock market volatility and its relationship to expected returns for major European stock markets. They found no relationship between stock market volatility and expected returns. However, none of the above studies investigated seasonal patterns in stock market volatility. Although there is a wide range of studies examining the seasonality patterns in average returns a limited set of studies examine these effects in the conditional volatility.

Fama (1965) reported the earliest evidence that the mean and variance of return distributions vary by day of the week. Ho and Cheung (1994) found that stock return variances of several Asia-Pacific markets are heterogeneous across days of the week. Berument and Kiyimaz (2001) showed that volatility varies by day of the week in the S&P 500 index. Their study assumes a GARCH specification under a normal distribution in the errors. Kiyimaz and Berument (2003) examined the day of the week effect on the mean, conditional volatility and transaction volume in major global equity indices assuming a GARCH specification under a normal error distribution. They found evidence of variation in return distributions by day of the week. Choudhry (2000) uses a GARCH specification where the error term follows a conditional Student-t distribution and finds evidence of the day of the week effect in mean and conditional variance for seven Asian emerging equity markets. Baker et al. (2008) examine the day of the week effect in both the mean and volatility in the S&P/TSX composite price index from the Toronto Stock Exchange using a GARCH specification under various error distribution assumptions. They find that the effect is sensitive to the error distributional assumptions.

Some empirical studies show that the financial time series have fatter tails than the normal distribution and exhibit volatility clustering. However, almost all previous studies ignore these stylized facts and uses standard methods such as ANOVA to test for equality of means or F and t tests on OLS regression with dummy variables to test for significance of the seasonality effects. This casts doubt on the reliability of results given that normality is one of the basic assumptions of these tests.
In this paper we allow for time varying conditional volatility and we consider at the same time a range of seasonality effects in the return and conditional variance equations on the regression analysis. We examine the robustness/instability of seasonality effects in models specified under different error distributions and also consider descriptive separated from explanatory models.

Data and Initial Statistical Tests

The data employed in this study are daily closing prices from the French stock market over the estimation period December, 3, 1990 to December, 30, 2009. The long-term market index was obtained from the Paris Stock Exchange - Euronext Paris. We use the CAC-40 index which is the main index and is based on 40 of the largest companies in terms of market capitalization. The constituent stocks of the index are the 40 most representative stocks in term of free-float adjusted capitalisation and turnover and the weighting scheme of the index is based on free-float adjusted market capitalisation. In the Euronext - Paris the index is available in terms of “net return” and “total return”, where the later incorporates a special “avoid fiscal” tax credit which takes into account the reinvestment of ordinary gross amount of dividends declared by companies in the index. For comparability with other studies our analysis is based on the “net return” index. The series of daily market returns are calculated as the continuously compounded returns where:

\[ r_t = \ln(P_t / P_{t-1}) \times 100 \]

\( r_t \) is the daily return in day \( t \) and \( P_t \) is the index level at the end of day \( t \).

Table 1 reports sample statistics for the CAC-40 return series over the full period and two sub periods related to the pre and introduction (December, 3, 1990 to December, 28, 2001) and post introduction period of the euro (January, 3, 2002 to December, 31, 2009). Table 1 contains statistics testing the null hypotheses of independent and identically distributed normal variates. The descriptive statistics for the index return series are, among others, the mean, standard deviation, skewness, excess kurtosis, first three-order autocorrelation coefficients, and the Ljung-Box Q(10) for the standardized residuals and the squared standardized residuals.

There is strong evidence, in all periods, against the assumption that returns are normally distributed. The evidence indicates significantly fatter tails than does the stationary normal distribution for each period. The skewness coefficient rejects the symmetric distribution null hypothesis only in the first sub period. The Jarque-Bera statistic and the comparison of the empirical distribution (Lilliefors statistic) with the theoretical one also reject the null hypothesis of normality of daily returns. The independence assumption for the \( T \) observations in each period is tested by calculating the first three order autocorrelation coefficients. Using the usual approximation of \( 1/\sqrt{T} \) as the standard error of the estimate, the statistics for the full period reject the second and third order zero correlation null hypothesis at the 5 and 1% level. Although not reported here, the autocorrelation function (ACF) from lag 1 to lag 40 in full period shows some small but significant autocorrelations at the 5% level. In the first ten lags the returns exhibit, mostly, negative autocorrelation. These significant coefficients are likely a result of the nonsynchronous trading effect.
The Ljung-Box Q(10) statistic for the cumulative effect of up to tenth-order autocorrelation in the standardized residuals exceeds the 1% critical value from a $\chi^2_{10}$ distribution for all three periods. The Ljung-Box Q(10) statistic on the squared standardized residuals provides us with a test of intertemporal dependence in the variance. The statistics for all three periods reject the zero correlation null hypotheses. That is, the distribution of the next squared return depends not only on the current return but on several previous returns. These results clearly reject the independence assumption for the time series of daily stock returns. Finally, Augmented Dickey-Fuller and Phillips-Peron tests reject the null hypothesis of a unit root and we conclude that the CAC-40 returns series over the full period and sub periods is stationary and suitable for a regression-based analysis.

The ADF test reported is performed with an intercept and an optimal lag structure according to the Akaike Information Criteria.

At a first stage we use parametric and nonparametric tests to examine for the existence of differences in average returns and volatility within returns categories of seasonality effects. Since the statistics in Table 1 show a non-normal distribution, the Brown and Forsythe (1978) test is used to test for the equality of variances which is robust to departures from normality. Although we could have used the Levine test, the Brown-Forsythe test is more robust when groups are unequal in size and the normality and equal variances are not verified. This test estimates whether more than two groups are homoscedastic. The Brown and Forsythe test statistic is the F statistic resulting from a one-way analysis of variance on the absolute deviations from the median.

Let be the th observation in the th group and let be the sample median for the th group, and let Brown and Forsythe’s test is to reject the null hypothesis of equal variances between groups if . represents the quantile of order of distribution and the level of significance of the test. To test for equality of mean returns across return categories of seasonality effects we use the Welch (1951)’s ANOVA modified F-test which accounts for the unequal variances, the standard ANOVA F-test and the nonparametric Kruskal-Wallis (KW) test. The test statistics for equality of means and variances are reported in Table 2.

Regarding tests for the homogeneity of the variance this statistic reveals significant differences across all seasonality effects.

### Table 1

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Full period</th>
<th>First sub-period</th>
<th>Second sub-period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>4822</td>
<td>2774</td>
<td>2048</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0274</td>
<td>0.045263</td>
<td>0.0032</td>
</tr>
<tr>
<td>S. deviation</td>
<td>1.4166</td>
<td>1.2815</td>
<td>1.5814</td>
</tr>
<tr>
<td>Maximum</td>
<td>10.594</td>
<td>6.8080</td>
<td>10.594</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.0392</td>
<td>-0.1862***</td>
<td>0.0855</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>7.8770***</td>
<td>5.2695***</td>
<td>8.8815***</td>
</tr>
<tr>
<td>JB test</td>
<td>4780***</td>
<td>610***</td>
<td>2954***</td>
</tr>
<tr>
<td>Empirical Distribution Test</td>
<td>0.0574***</td>
<td>0.0371***</td>
<td>0.0783***</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>-0.015</td>
<td>0.035*</td>
<td>-0.059***</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>-0.032**</td>
<td>-0.039**</td>
<td>-0.026</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>-0.062***</td>
<td>-0.036*</td>
<td>-0.080***</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>56.04***</td>
<td>23.86***</td>
<td>65.327***</td>
</tr>
<tr>
<td>Residual</td>
<td>2453.66***</td>
<td>362.98***</td>
<td>1326.2***</td>
</tr>
<tr>
<td>ADF unit root test</td>
<td>-11.45***</td>
<td>-21.17***</td>
<td>-7.599***</td>
</tr>
<tr>
<td>P-P unit root test</td>
<td>-70.89***</td>
<td>-50.88***</td>
<td>-48.799***</td>
</tr>
</tbody>
</table>

JB test: Jarque-Bera test for a normal distribution. Empirical Distribution Test is a goodness-of-fit test that compares the empirical distribution of daily returns with the normal theoretical distribution function. The value reported is the Lilliefors statistic. , , are the first three autocorrelations coefficients. Asterisks indicate significance at the 10%*, 5%** and 1% *** levels.
Asterisks indicate significance at the 10%*, 5%** and 1%*** levels. Sample period spans from Monday 3 December 1990 to Friday 30, December 2009. The test for equality of variances is the Brown-Forsythe (1974) test. The test statistics for the equality of means are the Welch (1951)’s modified ANOVA F-statistic, the ANOVA F-statistic and the Kruskal-Wallis statistic.

Regarding tests for the equality of means, we cannot reject the identical mean null hypothesis throughout days of the week and months of the year. For these two effects the results in the parametric and non-parametric statistics are consistent. For the TOM effect test results suggest that differences in means are significant at 5% in the Welch F-test and significant at the 1% level in the other two tests.

Regarding the holiday effect, results provided by parametric and non-parametric tests are consistent, suggesting that mean returns are significantly different at the 1% level across the three return categories. In sum, the Brown-Forsythe test rejects the homogeneity of volatility in all seasonality effects and the tests for the equality of means suggest differences in means returns in the TOM effect and across return categories of holiday effect.

### Seasonality Patterns in Mean Return with Different Distributional Assumptions

We consider at the same time the various seasonality effects and we start by estimating the effects in the following return equation using the OLS method:

\[
    r_t = \alpha_0 + \sum_{i=1}^{4} \alpha_i D_i + \sum_{j=1}^{11} \gamma_j M_j + \beta_1 \text{Pre}H_t + \beta_2 \text{Post}H_t + \sum_{m=1}^{11} \delta_m r_{t-m} + \varepsilon_t
\]

Where \( r_t \) is the return on day \( t \), \( D_i \) is a dummy variable taking a value of one for day \( i \) and zero otherwise (where \( i = 1, 2, 3, 4 \) ) and the reference category is Monday, \( M_j \) is a dummy variable taking a value of one for month \( j \) and zero otherwise (\( j = 1, 2, ..., 11 \)), the reference category is January, \( D_{TOM} \) is a dummy variable for the TOM period taking a value of one for TOM trading days and zero otherwise, PreH \(_t\) and PostH \(_t\) are dummy variables taking a value of one for a trading day preceding (following) a public holiday, respectively, and zero otherwise, \( r_{t-m} \) is the lagged return of order \( m \) and \( \varepsilon_t \) is the random error term of the regression assumed to be independently normally distributed with a zero mean and constant variance. Each coefficient of the regression is interpreted as follows. The intercept term, \( \alpha_0 \), is the mean return on a Monday in January, not included in the TOM period and which is not immediately before or after a public holiday. We interpret each coefficient for the dummy variables as its relative excess return to the intercept term. Eq.(4) attempts to simultaneously take into account all the above suggested seasonality patterns and allows partial tests of interactions between effects.

We base the choice of the lag length (\( k \) ) on the lowest Akaike information criterion (AIC). To remove the linear dependence in the return series we estimate an autoregressive model AR(\( k \) ) that minimizes the AIC. Then we estimate eq.(4) with this lag length and retest the resulting residuals from this equation for possible non-captured linear dependence. Lo and Mackinlay (1990) show that the non-synchronous trading causes linear dependence in the observed index returns but this effect is much less pronounced in price indices constituted by very liquid stocks. The Aikea’s criterion suggests an autoregressive model of order 5 to estimate the returns series.

### Table 2

<table>
<thead>
<tr>
<th>Seasonality effect</th>
<th>Number of categories</th>
<th>Tests of equality of variances: F-statistic</th>
<th>Tests of equality of means: Welch F-statistic</th>
<th>ANOVA F-statistic</th>
<th>KW statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day-of-the-week</td>
<td>5</td>
<td>2.8798**</td>
<td>0.3826</td>
<td>0.3890</td>
<td>0.6443</td>
</tr>
<tr>
<td>Month-of-the-year</td>
<td>12</td>
<td>4.7365***</td>
<td>0.9491</td>
<td>0.9690</td>
<td>9.8199</td>
</tr>
<tr>
<td>Turn-of-the-month</td>
<td>2</td>
<td>5.0989***</td>
<td>6.5179**</td>
<td>6.6609**</td>
<td>9.4670***</td>
</tr>
<tr>
<td>Holiday</td>
<td>3</td>
<td>4.2540***</td>
<td>10.0514***</td>
<td>5.9142***</td>
<td>14.1847***</td>
</tr>
</tbody>
</table>

Monteiro J. Sensitivity of seasonality effects on mean and conditional volatility to error distributional assumptions: evidence from French stock market. ECORFAN Journal-Mexico 2012, 3-8: 613-632
Eq.(4) assumes that residual terms are normally distributed with a constant variance. The estimated coefficients, the standard errors of the parameters and diagnostic statistics of eq.(4) are reported in Table 5. Table 5 only reports variables whose coefficients are significant. The standard errors of the OLS regression are corrected by the autocorrelation and heteroskedasticity consistent covariance estimator of Newey–West. As expected, due to the stylized facts of financial time series assumptions of normality and constancy of variance are rejected by the Jarque-Bera and the ARCH LM tests. The result from the ARCH LM test statistic indicates a time varying conditional heteroscedasticity in the CAC-40 daily index returns. The cumulative effect of autocorrelation coefficients of residuals up to twentieth-order is insignificant. However, Ljung-Box statistics for the cumulative effect up to thirtieth and fortieth-order autocorrelation in the residuals are significant indicating that the AR(5) model is not able to capture linear dependence at high lagged orders in the return series. Figure 1 shows the q-q plot of the standardized residuals of Eq.(4).

Figure 1

The Student-t, GED and DED are heavy-tailed distributions with positive excess kurtosis relative to a normal distribution, which has excess kurtosis of 0. Excess kurtosis is \( 6/(df - 4) \), where \( df \) = degrees of freedom, for the Student-t distribution and 3 for DED. For the GED distribution, the value of the shape parameter determines the thickness of the tail. When this parameter has a value less than 2, the distribution is thick tailed. When the values are 1 and 2, the result is DED and Normal distribution, respectively.

Thus, we analyze how the choice of error distribution affects seasonality patterns. Baillie and Bollerslev (1989) show that a GARCH (1,1) model provides a parsimonious fit for stock return series. Based on the selection criteria we chose an EGARCH (1,1) model after examining alternative models and combinations such as GARCH (1,1), GARCH (1,2), TGARCH (1,1), TGARCH (1,2) and EGARCH (1,2).

Since the null hypothesis of no ARCH effects in residuals of OLS regression is rejected, we examine the effect of assuming a time varying variance in seasonality patterns. We estimate Eq.(4) with a GARCH-type model with only the dummy variables in the conditional mean equation. We chose the best GARCH type model from the GARCH(p,q), TGARCH(p,q) and EGARCH(p,q) variations that best fit the daily index returns on the basis of Maximum Loglikelihood and AIC criteria, where \( p \) and \( q \) are the lag orders of the residuals and conditional variance in the variance equation. Empirical studies modeling the conditional volatility with GARCH type models generally assume a normal error distribution. As figure 1 shows the tails of the residuals of OLS regression are fatter than the normal distribution and it becomes appropriate to use a distribution with fatter tails. To capture fatter tails in the return series we use three distributions proposed by Nelson (1991) that better fit financial time series - Student–t, GED and DED error distributions.

Fig. 1 also rejects the normality assumption and shows that the tails of the residuals of OLS regression are fatter than the normal distribution.
Based on the leverage effects noted in Black (1976) and French et al. (1987), Nelson (1991) proposed the exponential GARCH (1,1) model

$$\ln \sigma_t^2 = \omega + \beta \ln \sigma_{t-1}^2 + \alpha \left( \frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \left( \frac{2}{\pi} \right)^{0.5} \right) + \phi \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}$$

In this formulation the conditional variance is an exponential function of the previous conditional variance and standardized unexpected return.

If $\alpha > 0$, then conditional volatility tends to increase (decrease) when the absolute value of the standardized unexpected return is larger (smaller). A positive $\alpha$ represents the empirical observation that large (small) price changes tend to follow a large (small) price change.

This is volatility clustering. An asymmetric effect occurs when an unexpected decrease in price resulting from bad news increases volatility more than an unexpected increase in price of similar magnitude, following good news.

This phenomenon has been attributed to the “leverage effect”: bad news lowers stock prices, increases financial leverage, and increases volatility. If $\phi < 0$, then conditional volatility tends to rise (fall) when the standardized unexpected return is negative (positive). Thus, this specification is expected to capture a large amount the skewness and leptokurtosis.

The estimated coefficients and standard errors of the parameters in the mean and conditional volatility equations of the EGARCH(1,1) model for the normal, Student’s t, GED and DED error distributions are reported in Table 3. Several relevant observations emerge from the results in Table 3.

First, our evidence supports the existence of the TOM effect with TOM trading days having a significantly higher average daily return. For the EGARCH (1,1) model with a normal error distribution, the average return for the TOM trading days is 0.12% higher than that for non-TOM trading days and the average daily return on trading days in this period is not significantly different from zero.

The excess average return of TOM trading days is significant at the 1% level. The results also support the existence of the holiday effect. When compared to ordinary trading days (days that do not precede or follows a holiday), the excess average return in the pre-holidays is 0.29% higher while for the post-holidays the excess average return is 0.31% higher being these excess returns significant at the 1% level.

The EGARCH with DED distribution also reports that the average return on August differs significantly from January at the 10% level and is about 0.10% less and the average return in January is not statistically different from zero.

Except for the DED distribution that reports a significant difference in average returns in August there is no reliable evidence to suggest the existence of the day-of-the-week or month-of-the-year effects.
TOM and holiday effects are significant in all error distributions despite the magnitude of their coefficients vary and average daily return on pre-holiday being significant at 1% level under the normal but are significant at 5% level under the Student’s t and GED distributions. In general, given results in Table 5, the examined seasonality effects do not depend on the assumptions of the error distributions. The significant effects shown under normal distribution tend to be robust under the other error distributions. However, there is the question of which of the four error distributions is more appropriate to capture the stylized facts of financial time series with seasonality effects included. To answer this question we examine which of the four EGARCH models best fits the data series. Although not shown, the q-q plots of standardized residuals of the four distributions show a better fit of distributions with fatter tails (Student's t and GED).

Since these plots do not help determine the best model, we make the decision based on the two model selection criteria: Maximum Log-Likelihood and AIC. Table 5 indicates that the two model selection criteria choose the model with a Student-t error distribution. The second best model is the GED distribution followed by the model with the normal distribution. The EGARCH model with the DED error distribution is the one with the worst fit to the data series. Thus, these results suggest that examining seasonality patterns, individually or jointly, assuming a normal error distribution may be inappropriate.

Thus, the reported significant effects of the TOM and holiday effects in the mean equation are not very sensitive to assumptions about the distribution of errors.

Table 3

This table reports results of the EGARCH(1,1) for the normal, Student’s t, GED and DED error distributions. The estimated model is the AR(5)-EGARCH(1,1) with the day-of-the-week, month-of-the-year, TOM and holidays effects only included in the mean equation. Table only reports dummy variables with significant coefficients. *, **, *** Statistically significant at the 10%, 5% and 1% level, respectively.

Thus, the reported significant effects of the TOM and holiday effects in the mean equation are not very sensitive to assumptions about the distribution of errors.
In this study, however, allowing for time varying conditional variance of the errors in the four distributions, all models detect the existence of the TOM and holiday effects that are consistent in terms of significance with those obtained in the preliminary analysis of equality of means in Table 2 and consistent in the signal, significance and magnitude with those obtained in OLS regression.

The results of diagnostic tests in Table 3 show that the EGARCH specification using the four error distributions reduces the intertemporal dependence in the standardized residuals and squared standardized residuals. The Ljung-Box statistics up to lag 40 do not reject the null hypothesis of zero autocorrelation coefficients. The ARCH LM test up to lag 20 is not significant, indicating that the four EGARCH models are successful in modeling the conditional volatility.

The Jarque-Bera test for normality rejects the null hypothesis that the standardized residuals are normally distributed, indicating that none of the four models are able to capture most of leptokurtosis present in the data series. For the four models the coefficient on the natural logarithm of the lagged conditional variance, , is significantly positive and smaller than one. Cross-sectionally, magnitudes of this coefficient are similar and indicate a long memory (smoothing) in the conditional variance. The hypothesis that is confirmed in all models at the 1% level, supporting the existence of volatility clustering. Finally, the hypothesis that for a leverage effect is evidenced by all models in magnitude and statistical significance.

Seasonality Patterns in Mean Return and Conditional Volatility with Different Distributional Assumptions

To test for the day-of-the-week, month-of-the-year, turn-of-the-month and holiday effects in volatility, we introduce the corresponding dummy variables into the mean and conditional variance equations of EGARCH(1,1) (6)

Where the dummy variables and parameters of the EGARCH conditional variance equation are defined as previously. Each coefficient of the dummy variables in the conditional variance equation is interpreted as follows. The intercept term, , is the mean volatility on a Monday in January, not included in the TOM period and which is not immediately before or after a public holiday. Each coefficient for the dummy variables is interpreted as its relative excess volatility to the intercept term.

Table 4 reports results from the estimation of the EGARCH(1,1) model with dummy variables included in the mean and conditional variance equations for normal, Student’s t, GED and DED error distributions. The results show that some effects of seasonality in conditional mean and variance are sensitive to the assumption of the distribution. Similar to the results reported in Table 3, our evidence supports the presence of the TOM effect. For the normal distribution the average daily return in the TOM trading days is about 0.11% higher than that in the non-TOM trading days whose average return is not significantly different from zero. This pattern is shown in the other error distributions with a similar magnitude in the coefficients. The results also support the existence of the holiday effect.
In the normal distribution the average daily return in the pre-holiday and post-holiday is higher by 0.25% and 0.31%, respectively, when compared to the average daily return in the other trading days where the average is not significantly different from zero. These significant effects are also shown in the other error distributions. The DED distribution reports that the intercept term is positive and significant at 5% level and the average daily return in August is lower by 0.13% compared to the average return in January. Except for the above significant terms reported by the DED error distribution there is no evidence of the day of the week and month of the year effects in the remaining error distributions. TOM and holidays effects in the mean equation are sensitive to the assumption about the underlying distribution. Specifically, assuming a normal distribution, the average return in the TOM trading days is significant at the 5% level, being however significant at the 1% level in fatter distributions as evidenced by the GED and DED. Additionally, the average return in pre-holiday days is significant at 5% under the normal and Student-t distributions but it is significant at 10% and 1% under the GED and DED distributions, respectively, with a considerable variation in the magnitude of the coefficients. Concerning the post-holiday trading day, the average return is significant at the 5% level under the normal but is significant at the 1% level under the Student-t, GED and DED distributions. Thus, in the mean equation there is instability in the significance level and in the coefficient estimate magnitudes.

For the conditional volatility equation, the significant estimated seasonality effects are also sensitive to the error distribution assumed in the model.

The presence of the day of the week effect with significant coefficients under all error distributions and the holiday effect (post-holiday) is evident only under the Student-t distribution.

Under the normal distribution the variance dummy variable for Tuesday and Friday is significant at the 5% level and the average volatility in these two days is 0.38 and 0.27 lower than the average volatility on Monday, after controlling for the persistence effect, volatility clustering and the asymmetry effect in the volatility equation. When we consider the Student-t distribution, the variance dummy variable for Tuesday and Friday remains significant but changes occur in the significance level and magnitude of the estimated coefficients. In this case Tuesday dummy variable is significant at the 1% level. Under this distribution the post-holiday variance dummy variable also reveals significant at the 10% level and the average volatility in this trading day is 0.15 higher than the average volatility on trading days not preceding or following public holidays. For the model with GED distribution the variance dummy variables for the Tuesday, Thursday and Friday are significant at the 1, 10 and 5% level and are, respectively, 0.35, 0.16 and 0.26 lower than average volatility for Monday. Concerning the model with the DED distribution only the variance dummy variable for Tuesday revealed significant at the 1% level and the volatility is 0.34 lower than the average volatility on Monday.

A striking result occurs in the DED distribution where the estimation process provides the more efficient parameter estimates (lowest standard errors) for the mean equation and, simultaneously, the less efficient estimates (highest standard errors) for the conditional variance equation among all error distributions.
In the preliminary analysis for variance homogeneity on the various seasonality effects it resulted that the Brown-Forsythe test statistic rejected the null hypothesis of homogeneity of variance across the return categories on all seasonality effects. However, when we allow for time varying conditional variance on the errors to detect the existence of differences in volatility across the various effects, differences only reveal significant on the day-of-the-week effect and only between a few days.

Table 4 also reports the results of the diagnostic statistics that examine whether all EGARCH models are correctly specified. As shown in Table 3, the EGARCH specification for the four error distributions cancels the intertemporal dependence in standardized residuals and squared standardized residuals. The Ljung-Box statistics up to lag 40 could not reject the zero autocorrelation coefficients null hypothesis in standardized residuals. The ARCH LM test statistic up to lag 20 could not reject the null hypothesis of serial independence in squared standardized residuals indicating that the four EGARCH models were successful in modeling the conditional volatility. The Jarque-Bera test for normality rejects the null hypothesis that the standardized residuals are normally distributed. Concerning the model that best fit the data series, results indicate that the model with a Student’s-t error distribution outperforms others distributions based on the maximum log-likelihood and AIC model selection criteria.

In sum, the estimated and significant parameters of the seasonality effects in the mean and conditional variance equations are sensitive to the error distribution that is assumed in the EGARCH specification, either in magnitude, significance and significance level.

<table>
<thead>
<tr>
<th>Table 4</th>
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<tr>
<td>This table reports results of the EGARCH(1,1) for the normal, Student’s -t, GED and DED error distributions. The estimated model is the AR(5)-EGARCH(1,1) with the day-of-the-week, month-of-the-year, TOM and holidays effects included in the mean and conditional variance equations. Table only reports dummy variables with significant coefficients. AIC – Akaike Information Criteria. * , **, *** Statistically significant at 10%, 5% and 1% level, respectively.</td>
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Forecast Performances in Mean Return and Volatility

In order to test the importance of the seasonality effects the previously estimated EGARCH models were used for out-of-sample one-step-ahead forecasts for 2010 in returns and conditional volatility. This gives 258 forecasts. The measure of forecast accuracy used is the root mean square error (RMSE), which penalizes large errors in either direction. The results are reported in Table 5 for the estimated models with different error distributions.

The forecasts for returns and conditional volatility are firstly performed in models including only the seasonality effects in the mean equation and then performed in models including either seasonality effects in the mean and conditional volatility equations. As additional references in the forecast accuracy we also consider the autoregressive model AR(k=5) and the random walk model in forecasting returns, where denote the historical mean return. Andersen and Bollerslev (1998) argue that squared daily returns provide a very noise proxy for the ex post volatility and a much better proxy for the day’s variance would be to compute the volatility for the day from intra-daily data. As we have no intra-day prices taken at hourly intervals, we use as the proxy for the actual volatility the variance of six intra-day returns calculated as (close-open prices), (close-maximum prices), (close-minimum prices), (maximum-minimum prices), (maximum-open prices), (minimum-open prices). In performing forecasts for returns and conditional volatility, all models, except the random walk model, include autoregressive terms of returns in the mean equation.

<table>
<thead>
<tr>
<th>Return Volatility</th>
<th>RMSE</th>
<th>Rank</th>
<th>RMSE</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seasonality effects included in mean equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Random Walk</td>
<td>1.4668</td>
<td>1</td>
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<td>-</td>
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<td>AR (k=5)</td>
<td>1.4812</td>
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<td>-</td>
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<td>LLS</td>
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<td>-</td>
<td>-</td>
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<td>2.2706</td>
<td>1</td>
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<tr>
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<td>2.2738</td>
<td>3</td>
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<tr>
<td>EGARCH – GED</td>
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<td>3</td>
<td>2.2769</td>
<td>2</td>
</tr>
<tr>
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<td>2.4192</td>
<td>7</td>
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<tr>
<td>Seasonality effects included in mean and conditional volatility equations</td>
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<tr>
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<tr>
<td>EGARCH – DED</td>
<td>1.4733</td>
<td>9</td>
<td>2.4434</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 5

RMSE – Root Mean Square Error, Rank = 1 for smallest RMSE.

From table 5 it can be seen that the random walk model gives better forecasts for returns and the OLS model performs worst among all the estimated models. Within the set of EGARCH models for the various error distributions, those specifications that do not include seasonality effects in the conditional volatility equation gives better forecasts and, among these, the EGARCH with normal distribution provides the best forecasts followed by GED distribution. DED distribution provides the worst forecasts with and without seasonality effects included in conditional volatility equation. Concerning volatility forecasts, EGARCH models with seasonality effects not included in the conditional variance equation provides best forecasts with the normal performing best and the DED performing worst. Thus, albeit some seasonality effects are significant in the return and conditional volatility equations in the above descriptive models, these effects are not useful in explanatory models and do not introduce predictive ability against the random walk model.
Summary and Conclusion

In this paper we investigate the day-of-the-week, month-of-the-year, TOM and holiday effects in return and conditional volatility. We examine the sensitivity in inference that might occur when using different distributional assumptions for the error terms in GARCH modelling. We examine daily time series data of the French CAC-40 index. The four different error distributions are the normal, Student’s-t, generalized error distribution and double exponential distribution. We test whether inferences drawn from statistical test are robust to different error distributions. We also examine the usefulness of the significant estimated seasonalties in the return and volatility equations to forecast out-of-sample return and volatility.

We consistently find the presence of the turn-of-the-month and holiday effects in return equations for the French CAC-40 stock index using a EGARCH (1,1) model. We find evidence that the average return in the TOM period and in the pre- and post-holiday days are significantly higher than the average return in the other trading days. No significant coefficient of the day-of-the-week and month-of-the-year was found except the August dummy variable in the model with the DED distribution. We show that conditional volatility only varies with some days of the week but results are not consistent across different error distributions. In sum, we show that many of the expected seasonality effects are small and not significant, the significant dummy variables in return and conditional volatility are sensitive to the error distribution that is specified under the EGARCH descriptive model and the Student’s-t distribution best describes stock index returns.

We examine if the significant dummy variables found in the return and conditionally volatility equations in descriptive models are useful for forecasting out-of-sample the return and volatility. Results show that the in-sample significant effects do not add forecast improvements against the random-walk model.

Our conclusion, based on this evidence, is that significant effects obtained from studies of seasonality patterns may be fragile. Although some significant effects could manifest in the in-sample period, the inference is instable to different error distributions and the estimated significant effects do not have forecast ability for out-of-sample forecast for returns and volatility. The above evidence adds to the literature that cast doubts on the economic significance of the seasonality effects.

References


