Parental investment in children’s education

Jaime Andrés Sarmiento Espinel
jaime.sarmiento@unimilitar.edu.co

Universidad Militar Nueva Granada

4th Annual Congress ECORFAN
May 16, 2013
Does a change on the decision process between parents affect resources directed to children’s welfare?

- Empirical evidence of a relation between factors that may influence the distribution of power between household members and intrahousehold allocations.
Motivation

Does a change on the decision process between parents affect resources directed to children’s welfare?

- Empirical evidence of a relation between factors that may influence the distribution of power between household members and intrahousehold allocations.

- In general terms, particularly for Mexico, there is no evidence of household decision process that considers at the same time:
  - changes in bargaining power distribution in favor of either parent, and
  - both parental time and expenditures on children’s education.
Motivation

Does a change on the decision process between parents affect resources directed to children’s welfare?

- Empirical evidence of a relation between factors that may influence the distribution of power between household members and intrahousehold allocations.

- In general terms, particularly for Mexico, there is no evidence of household decision process that considers at the same time:
  - changes in bargaining power distribution in favor of either parent, and
  - both parental time and expenditures on children’s education.

- Although some of the empirical evidence suggests that favorable changes in mothers’ bargaining power benefit children welfare, the lens used to interpret this type of observed behavior is crucial to policy issues.
Households in the economic literature

**Unitary**

- Under some assumptions, the household members’ utilities and budget constraints can be systematically added.
  - A household acts as a single rational individual.
- Several authors have offered evidence against its main implications (e.g. income pooling hypothesis).

**Collective**

- Household members with heterogeneous preferences.
- Disagreements are resolved by means of their relative bargaining power.
- It assumes that the intra-household decisions are Pareto efficient.
- It would allow to propose policies that affect directly the bargaining power and to target a specific household member.
The Current State of Play

- Blundell, Chiappori, and Meghir (2005) -from now on BCM- develop a collective model of household labor supply where:
  - Both parents obtain utility (although not necessary at the same degree) from the welfare of their offspring.
  - Children’s utility can be augmented by means of specific expenditures and parental time.
  - Increments on the bargaining power of a parent result in more resources directed to children only when her/his willingness to pay for children’s welfare is more sensitive to increases in her/his private consumption.

Until the moment, the only known empirical application to this model has been made by Cherchye, De Rock, and Vermeulen (2010, 2012) -from now on CRV- with Dutch data.
Blundell, Chiappori, and Meghir (2005) -from now on BCM- develop a collective model of household labor supply where:

- Both parents obtain utility (although not necessary at the same degree) from the welfare of their offspring.
- Children’s utility can be augmented by means of specific expenditures and parental time.
- Increments on the bargaining power of a parent result in more resources directed to children only when her/his willingness to pay for children’s welfare is more sensitive to increases in her/his private consumption.

Until the moment, the only known empirical application to this model has been made by Cherchye, De Rock, and Vermeulen (2010,2012) -from now on CRV- with Dutch data.
Objective

- Examine the effects of changes in the distribution of power between parents over intra-household resource allocation directed to children’s welfare, particularly education.
Examine the effects of changes in the distribution of power between parents over intra-household resource allocation directed to children’s welfare, particularly education.

BCM’s model is identified using a particular parametrization and an estimation strategy based on CRV.
Examine the effects of changes in the distribution of power between parents over intra-household resource allocation directed to children’s welfare, particularly education.

BCM’s model is identified using a particular parametrization and an estimation strategy based on CRV.

The sample is drawn from the second wave of the Mexican Family Life Survey (MxFLS-2) and focused on Mexican nuclear families with only children under 15 years and at least one school-age child, where both parents work.
Objective

- Examine the effects of changes in the distribution of power between parents over intra-household resource allocation directed to children’s welfare, particularly education.
  - BCM’s model is identified using a particular parametrization and an estimation strategy based on CRV.
  - The sample is drawn from the second wave of the Mexican Family Life Survey (MxFLS-2) and focused on Mexican nuclear families with only children under 15 years and at least one school-age child, where both parents work.
  - The impact of changes of some factors that could influence the Pareto weights are used to assess how resources directed to children’s education respond to it.
Collective labor supply with children and home production model of BCM

\[
\begin{align*}
\max_{L^m, L^f, C^m, C^f, K, h^m_K, h^f_K} & \quad \lambda U^m (L^m, C^m, u^K) + (1 - \lambda) U^f (L^f, C^f, u^K) \\
\text{s.t.} & \quad w^m L^m + w^f L^f + C^m + C^f + K = (w^m + w^f) T + y \\
& \quad L^i + h^i_W + h^i_K = T, \quad i = m, f \\
& \quad u^K (K, h^m_K, h^f_K) = u^K
\end{align*}
\]

- \((\lambda^m, \lambda^f) = (\lambda, 1 - \lambda)\)
- The Pareto weight \((\lambda^m, \lambda^f)\) reflects the relative power of \((m, f)\) in the household.
- It is assumed that \(\lambda\) is a function of wages, nonlabor income, and some distribution factors \((w^m, w^f, y, z)\).
Two stage process problem

1\textsuperscript{st} stage: Parents agree on

(a) children’s utility expenditure and production, and

$$\min_{K, h^m_K, h^f_K} e^K = w^m h^m_K + w^f h^f_K + K \quad \text{s.t.} \quad u^K(K, h^m_K, h^f_K) = u^K$$
Two stage process problem

1st stage: Parents agree on

(a) children’s utility expenditure and production, and

\[
\min_{K, h^m_K, h^f_K} e^K = w^m h^m_K + w^f h^f_K + K \quad \text{s.t.} \quad u^K (K, h^m_K, h^f_K) = u^K
\]

(b) how to allocate among them the residual nonlabor income.

\[
\max_{\phi^m, \phi^f, u^K} \lambda V^m (w^m, \phi^m, u^K) + (1 - \lambda) V^f (w^f, \phi^f, u^K) \quad \text{s.t.} \quad \phi^m + \phi^f + e^K = y
\]
Two stage process problem

1\textsuperscript{st} stage: Parents agree on

(a) children’s utility expenditure and production, and
\[
\min_{K, h_K^m, h_K^f} e^K = w^m h_K^m + w^f h_K^f + K \quad \text{s.t.} \quad u^K (K, h_K^m, h_K^f) = u^K
\]

(b) how to allocate among them the residual nonlabor income.
\[
\max_{\phi^m, \phi^f, u^K} \lambda V^m (w^m, \phi^m, u^K) + (1 - \lambda) V^f (w^f, \phi^f, u^K)
\]
\[
\quad \text{s.t.} \quad \phi^m + \phi^f + e^K = y
\]

2\textsuperscript{nd} stage: Each parent separately chooses their private consumption subject to their corresponding budget constraint.
\[
\max_{L^i, C^i} U^i (L^i, C^i, \bar{u}^K) \quad \text{s.t.} \quad w^i L^i + C^i = w^i T + \phi^i
\]
Empirical application

Parametric specification

- 2\textsuperscript{nd} stage: Indirect utility functions of PIGLOG class:
  \[
  V^i (w^i, \phi^i, u^K) = \frac{\ln(w^i T + \phi^i) - (\alpha^i_1 + \alpha^i_2 u^K) \ln w^i}{(w^i)^{\beta^i}}, \quad i = m, f
  \]

- 1\textsuperscript{st} stage: Children’s utility production function has a CES form:
  \[
  u^K (K, h^m_K, h^f_K) = \left[\gamma^m (h^m_K)^{\rho} + \gamma^f (h^f_K)^{\rho} + \gamma^K K^{\rho}\right]^{1/\rho},
  \gamma^i > 0, \quad \gamma^m + \gamma^f + \gamma^K = 1, \quad \rho \leq 1, \quad \sigma = \frac{1}{1-\rho}
  \]

- Mother’s Pareto weight has the form:
  \[
  \lambda (w^m, w^f, y, z_1, z_2) = \frac{\exp(\Lambda_1 + \Lambda_2 (w^m/w^f) + \Lambda_3 y + \Lambda_4 z_1 + \Lambda_5 z_2)}{1 + \exp(\Lambda_1 + \Lambda_2 (w^m/w^f) + \Lambda_3 y + \Lambda_4 z_1 + \Lambda_5 z_2)}, \quad \lambda \in [0, 1]
  \]
The resulting system consists of five equations \((h^m_K, h^f_K, K, L^m, L^f)\) as functions of \((w^m, w^f, y, z_1, z_2)\).

\[
h^i_K = \left(\frac{\gamma^i}{w^i}\right)\sigma A^{-1/\rho} \left(\frac{y + (w^m + w^f)T}{A^{1/(1-\sigma)}} + \frac{C}{D}\right)
\]

\[
K = (1 - \gamma^m - \gamma^f)^\sigma A^{-1/\rho} \left(\frac{y + (w^m + w^f)T}{A^{1/(1-\sigma)}} + \frac{C}{D}\right)
\]

\[
L^i = \left(\alpha^i_1 + \alpha^i_2 u^K + \beta^i \left(\ln \left(-\lambda^i (w^j)^{\beta^j} A^{1/(1-\sigma)}\frac{D^{1/1-\sigma}}{w^i D}\right) - (\alpha^i_1 + \alpha^i_2 u^K) \ln w^i\right)\right) \left(\frac{-\lambda^i (w^j)^{\beta^j} A^{1/(1-\sigma)}}{w^i D}\right)
\]

where

\[
A = (\gamma^m)^\sigma (w^m)^{1-\sigma} + (\gamma^f)^\sigma (w^f)^{1-\sigma} + (1 - \gamma^m - \gamma^f)^\sigma
\]

\[
C = \lambda (w^f)^{\beta^f} + (1 - \lambda) (w^m)^{\beta^m}
\]

\[
D = \lambda (w^f)^{\beta^f} \alpha^m_2 \ln w^m + (1 - \lambda) (w^m)^{\beta^m} \alpha^f_2 \ln w^f
\]
The system is estimated using the Feasible Generalized Nonlinear Least Squares (FGNLS) estimator.

Robust standard errors are used to guard against possible misspecification of the variance matrix.
Data

- Estimation of the empirical model specified in the preceding subsection is information demanding (labor market, incomes, expenditures, time allocation).
- A survey that satisfies the information requirements is the Mexican Family Life Survey (MxFLS/ENNVIH for its abbreviation in Spanish).
- A subsample (158 households) is extracted from the second wave of the MxFLS (2005-2006).
  - Nuclear families with only children under 15 years and at least one school-aged child.
  - Both parents participate in the labor market.
## Descriptive statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mother</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h^m_K$ : Time devoted to children’s education (h/week)</td>
<td>2.981</td>
<td>4.552</td>
</tr>
<tr>
<td>$h^m_W$ : Time devoted to market work (h/week)</td>
<td>33.576</td>
<td>17.818</td>
</tr>
<tr>
<td>$w^m$ : Wage rate (MXN per hour)</td>
<td>36.200</td>
<td>64.025</td>
</tr>
<tr>
<td>Age</td>
<td>34.272</td>
<td>5.698</td>
</tr>
<tr>
<td>Years of education</td>
<td>9.949</td>
<td>4.072</td>
</tr>
<tr>
<td><strong>Father</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h^f_K$ : Time devoted to children’s education (h/week)</td>
<td>1.652</td>
<td>2.986</td>
</tr>
<tr>
<td>$h^f_W$ : Time devoted to market work (h/week)</td>
<td>47.766</td>
<td>16.535</td>
</tr>
<tr>
<td>$w^f$ : Wage rate (MXN per hour)</td>
<td>37.768</td>
<td>61.797</td>
</tr>
<tr>
<td>Age</td>
<td>36.994</td>
<td>7.627</td>
</tr>
<tr>
<td>Years of education</td>
<td>9.158</td>
<td>4.129</td>
</tr>
<tr>
<td><strong>K</strong> : Education expenses (MXN per week)</td>
<td>85.563</td>
<td>193.294</td>
</tr>
<tr>
<td><strong>y</strong> : Nonlabor income (MXN per week)</td>
<td>257.508</td>
<td>389.086</td>
</tr>
<tr>
<td><strong>z_1</strong> : Mother’s age difference w.r.t. father</td>
<td>-2.722</td>
<td>6.237</td>
</tr>
<tr>
<td><strong>z_2</strong> : Mother’s education difference w.r.t. father</td>
<td>0.791</td>
<td>4.261</td>
</tr>
<tr>
<td><strong>N_K</strong> : Number of children in school-age</td>
<td>1.899</td>
<td>0.823</td>
</tr>
</tbody>
</table>
## Estimation results (A)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Robust Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parent’s preferences‡</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Mother</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha^m_1$</td>
<td>0.7703**</td>
<td>0.3550</td>
</tr>
<tr>
<td>$\alpha^m_2$  (Children’s utility)†</td>
<td>-0.0027***</td>
<td>0.0004</td>
</tr>
<tr>
<td>$\tilde{\alpha}^m_2$</td>
<td>-5.9160***</td>
<td>0.1409</td>
</tr>
<tr>
<td>$\beta^m$</td>
<td>-0.0737</td>
<td>0.0544</td>
</tr>
<tr>
<td><strong>Father</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha^f_1$</td>
<td>-0.8844***</td>
<td>0.3076</td>
</tr>
<tr>
<td>$\alpha^f_2$  (Children’s utility)†</td>
<td>-0.0033***</td>
<td>0.0008</td>
</tr>
<tr>
<td>$\tilde{\alpha}^f_2$</td>
<td>-5.7000***</td>
<td>0.2479</td>
</tr>
<tr>
<td>$\beta^f$</td>
<td>0.1577***</td>
<td>0.0399</td>
</tr>
</tbody>
</table>

The expressions in braces refer to the objects that are related to the respective parameters

* $p<0.1$, ** $p<0.05$, *** $p<0.01$

† $V^i(w^i, \phi^i, u^K) = \frac{\ln(w^i T + \phi^i) - (\alpha^i_1 + \alpha^i_2 u^K) \ln w^i}{(w^i)^{\beta^i}}, \quad i = m, f$

‡ $\alpha^i_2 = -\exp(\tilde{\alpha}^i_2), \quad i = m, f$
### Estimation results (B)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Robust Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Children’s utility production‡‡</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma^m$ (Mother’s time)‡‡</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{\gamma}_1^m$</td>
<td>31.4583***</td>
<td>0.3463</td>
</tr>
<tr>
<td>$\tilde{\gamma}_2^m$</td>
<td>1.0956***</td>
<td>0.0118</td>
</tr>
<tr>
<td>$\gamma^f$ (Father’s time)‡‡</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{\gamma}_1^f$</td>
<td>29.5919***</td>
<td>0.1216</td>
</tr>
<tr>
<td>$\tilde{\gamma}_2^f$</td>
<td>-0.0012***</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\sigma$ (Substitution elasticity)</td>
<td>0.0499**</td>
<td>0.0242</td>
</tr>
<tr>
<td>$\rho$ (Substitution parameter)‡‡‡</td>
<td>-19.0493*</td>
<td>9.7290</td>
</tr>
<tr>
<td>$\tilde{\rho}$</td>
<td>-2.9982***</td>
<td>0.4853</td>
</tr>
</tbody>
</table>

The expressions in braces refer to the objects that are related to the respective parameters

* $p<0.1$, ** $p<0.05$, *** $p<0.01$

‡‡ $u^K (K, h^m_K, h^f_K) = \left[ \gamma^m (h^m_K)^\rho + \gamma^f (h^f_K)^\rho + \gamma^K K^\rho \right]^{1/\rho}$

‡‡ $\gamma^i = \frac{\exp(\tilde{\gamma}_1^i + \tilde{\gamma}_2^i N_K)}{1 + \exp(\tilde{\gamma}_1^i + \tilde{\gamma}_2^i N_K) + \exp(\tilde{\gamma}_1^j + \tilde{\gamma}_2^j N_K)}$, $i \neq j, i = m, f$

‡‡‡ $\rho = 1 - \exp(-\tilde{\rho})$
## Estimation results (C)

<table>
<thead>
<tr>
<th>Pareto weight†††</th>
<th>Estimate</th>
<th>Robust Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_1$</td>
<td>-0.6434</td>
<td>0.5193</td>
</tr>
<tr>
<td>$\Lambda_2$ (Mother’s relative wage)</td>
<td>0.4113***</td>
<td>0.0467</td>
</tr>
<tr>
<td>$\Lambda_3$ (Nonlabor income)</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\Lambda_4$ (Age difference w.r.t. father)</td>
<td>0.0130*</td>
<td>0.0073</td>
</tr>
<tr>
<td>$\Lambda_5$ (Education difference w.r.t. father)</td>
<td>0.0061</td>
<td>0.0086</td>
</tr>
</tbody>
</table>

The expressions in braces refer to the objects that are related to the respective parameters

* $p<0.1$, ** $p<0.05$, *** $p<0.01$

††† $\lambda (w^m, w^f, y, z_1, z_2) = \frac{\exp(\Lambda_1+\Lambda_2 (w^m/w^f)+\Lambda_3 y+\Lambda_4 z_1+\Lambda_5 z_2)}{1+\exp(\Lambda_1+\Lambda_2 (w^m/w^f)+\Lambda_3 y+\Lambda_4 z_1+\Lambda_5 z_2)}$
### Partial derivatives of the sharing rule and elasticities

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \frac{\partial \phi}{\partial \text{Variable}} )</th>
<th>Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mother (1)</td>
<td>Father (2)</td>
</tr>
<tr>
<td>Mother’s wage</td>
<td>-45.386***</td>
<td>6.112</td>
</tr>
<tr>
<td></td>
<td>(9.280)</td>
<td>(4.382)</td>
</tr>
<tr>
<td>Father’s wage</td>
<td>26.609**</td>
<td>-81.261***</td>
</tr>
<tr>
<td></td>
<td>(11.999)</td>
<td>(11.547)</td>
</tr>
<tr>
<td>Non-labor income</td>
<td>0.139</td>
<td>-0.112</td>
</tr>
<tr>
<td></td>
<td>(0.143)</td>
<td>(0.111)</td>
</tr>
<tr>
<td>Age diff. w.r.t. father</td>
<td>22.480*</td>
<td>-18.113</td>
</tr>
<tr>
<td></td>
<td>(12.511)</td>
<td>(11.932)</td>
</tr>
<tr>
<td>Education diff. w.r.t. father</td>
<td>10.579</td>
<td>-8.524</td>
</tr>
<tr>
<td></td>
<td>(14.706)</td>
<td>(12.501)</td>
</tr>
</tbody>
</table>

* p<0.1, ** p<0.05, *** p<0.01. Delta-method standard errors are in parentheses.
### Partial derivatives of the sharing rule and elasticities - Continued

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\frac{\partial \phi^i}{\partial \text{Variable}}$</th>
<th>Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mother (1) / Father (2)</td>
<td>Children’s utility production</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mother (7) / Father (8) / Expenses (9) / $u^K$ (10)</td>
</tr>
<tr>
<td>Mother’s wage</td>
<td>-45.386*** / 6.112</td>
<td>0.168 / 0.218* / 0.218* / 0.190</td>
</tr>
<tr>
<td></td>
<td>(9.280) / (4.382)</td>
<td>(0.136) / (0.124) / (0.124) / (0.138)</td>
</tr>
<tr>
<td>Father’s wage</td>
<td>26.609** / -81.261***</td>
<td>0.603*** / 0.553*** / 0.603*** / 0.580***</td>
</tr>
<tr>
<td></td>
<td>(11.999) / (11.547)</td>
<td>(0.123) / (0.117) / (0.123) / (0.117)</td>
</tr>
<tr>
<td>Non-labor income</td>
<td>0.139 / -0.112</td>
<td>0.127*** / 0.127*** / 0.127*** / 0.127***</td>
</tr>
<tr>
<td></td>
<td>(0.143) / (0.111)</td>
<td>(0.015) / (0.015) / (0.015) / (0.018)</td>
</tr>
<tr>
<td>Age diff. w.r.t. father</td>
<td>22.480* / -18.113</td>
<td>0.006 / 0.006 / 0.006 / 0.006</td>
</tr>
<tr>
<td></td>
<td>(12.511) / (11.932)</td>
<td>(0.010) / (0.010) / (0.010) / (0.010)</td>
</tr>
<tr>
<td>Education diff. w.r.t. father</td>
<td>10.579 / -8.524</td>
<td>-0.001 / -0.001 / -0.001 / -0.001</td>
</tr>
<tr>
<td></td>
<td>(14.706) / (12.501)</td>
<td>(0.002) / (0.002) / (0.002) / (0.002)</td>
</tr>
</tbody>
</table>

*p<0.1, ** p<0.05, *** p<0.01. Delta-method standard errors are in parentheses.
Effect of change in parents’ Pareto weight
Final remarks

- Under the collective approach, resources invested in child’s welfare depend not only of the household budget constraint but also of their parents’ individual preferences and their relative position in the decision making.
- BCM’s model is applied to a sample of Mexican nuclear families from the MxFLS-2.
- Based on CRV, the estimation strategy takes advantage of the two-stage representation of the collective model to construct a flexible functional specification for the observables.
The estimation results highlights some interesting findings for this particular sample.

- In the production of children’s utility by means of resources used in their education, parental time and expenditure are complementaries inputs.
- Bargaining power distribution depends significantly of mother’s relative wage and age difference with respect to father.
- Parents’ preferences are not separable from resources directed to children’s welfare.
For families with average characteristics in the sample, evidence that mothers care more for their children than fathers were not found. Maintaining all else equal, there are more resources directed to children’s education when a father has more power via his relative wage.
For families with average characteristics in the sample, evidence that mothers care more for their children than fathers were not found. Maintaining all else equal, there are more resources directed to children’s education when a father has more power via his relative wage.

Graphical illustrations of an exogenous change in mothers’ Pareto weight at the baseline are provided. Although mothers’ have a larger marginal willingness to pay than fathers for children’s utility, it is only when father’s power increases, but not when mothers have more power, that more resources are directed to children’s utility production.
Comments


Distribution factors (DFs)

- External factors that affect the decision process but not the individual preferences or the joint budget set (after controlling for total income).
- Household decisions could represent the outcome of some bargaining process between its members.
- The individuals compare the utility level achieved if they cooperate (the gains of living together) with the level obtained if she/he doesn’t cooperate.
- The non-cooperative utility of a member serves as a threat point to obtain more power in the bargaining process.
- Thus, changes in DFs shift the threat points and therefore the Pareto weights.
- Examples: control over economic resources (assets, unearned income, targeted transfers, ...), marriage market, legislation, skills and knowledge.
How does a change in the distribution of power affect resources directed to children?

- From the first-order conditions, the Samuelson conditions characterize the production of children's human capital:

\[
\lambda \frac{\partial V^m}{\partial \phi^m} = (1 - \lambda) \frac{\partial V^f}{\partial \phi^f} \Rightarrow \frac{\partial V^m / \partial u^K}{\partial V^m / \partial \phi^m} + \frac{\partial V^f / \partial u^K}{\partial V^f / \partial \phi^f} = \frac{\partial e^K}{\partial u^K}
\]

- Prop 1 BCM: If \( i \) such that \( \frac{\partial MWP_i^i}{\partial u^K} < 0 \) and \( \frac{\partial MWP_i^i}{\partial \phi^i} > 0 \), \( i = m, f \)

\[
\Rightarrow \frac{\partial u^K}{\partial \lambda^i} > 0 \quad \text{iff} \quad \frac{\partial MWP_i^i}{\partial \phi^i} > \frac{\partial MWP_j^j}{\partial \phi^j}
\]